Findings from four iterations of a first year problem solving course

THE UNIVERSITY OF NEW SOUTH WALES

Never Stand Still

UNSW Canberra

Tristram Alexander
School of Physical, Environmental and Mathematical Sciences
UNSW Canberra

Students and the School

The students…

First-year engineering students

Students from all over Australia.

Some international.

Clear career path on graduation.

Paid to be at University. Typically around 90% lecture attendance.

The engineering school…

Represented by an amorphous grouping of lecturers from School of Engineering and Information Technology at UNSW Canberra.

Background to the course

Engineering Computational Methods

2008…2012

Programming

Implementation of algorithms in Matlab

Outcome:
majority of students ended up hating Matlab

Engineering School requirements:
Students seem to be bad at solving problems, and particularly seem to struggle with mathematics, put together a course which fixes this.

Complications:

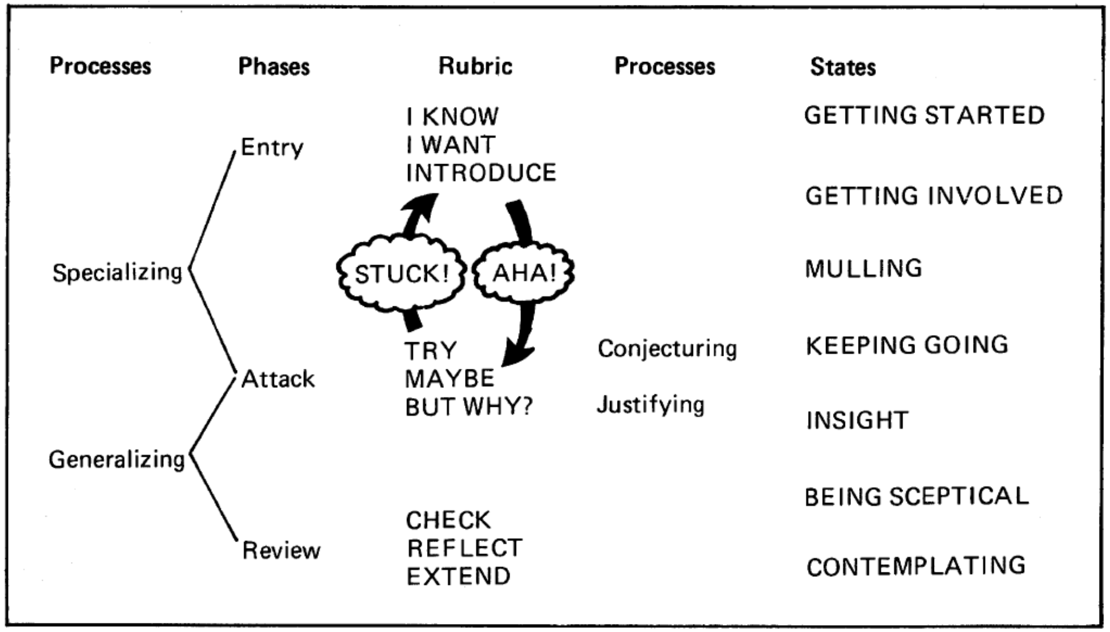
- Course should have some computational part;
- Students face many other types of courses, not just mathematics courses;
- No other courses will change.

In an ideal world…?

It would be good if:

- Students are interested in mathematics, and motivated intrinsically (e.g. seeking understanding) rather than extrinsically (e.g. to obtain a pass mark).
- Students work through e.g. Mason *et al*. and be proactive in developing their mathematical thinking.
- Other courses take similar approaches and also work to develop problem solving skills

But even if all this were true we would still face significant questions as teachers. What to do in class to promote problem solving skill development?



"Thinking Mathematically", 2nd Ed., J. Mason, L. Burton and K. Stacey

Implications of constraints

- Mathematical thinking at varied levels (needs scalable approach, not dogmatic prescription of what to do)
- Students focused on applied questions (which requires familiarity with a broad range of courses)
- Students focused on assessment (soft skills and process need to be somehow assessed)
- Students time-poor and risk-averse (favouring a traditional class over anything markedly different)

Overview of the course development

Engineering Problem Solving

2013…2014

- Programming
- McMaster Problem Solving Program

Specific skill development

2015

- Algorithmic thinking
- Psychology
- Mathematical thinking

Kahneman: awareness of biases which affect problem solving
Alcock: how to study maths

2016

- Algorithmic thinking
- Mathematical problem solving

Zeitz: Practice in solving problems using specific techniques.

1st iteration: McMaster Problem Solving Program

Focus on explicit skill development

For each course area:

- Define skills
- Motivate use & importance
- Provide Pre-test, Objectives, Targets
- Outline route ahead

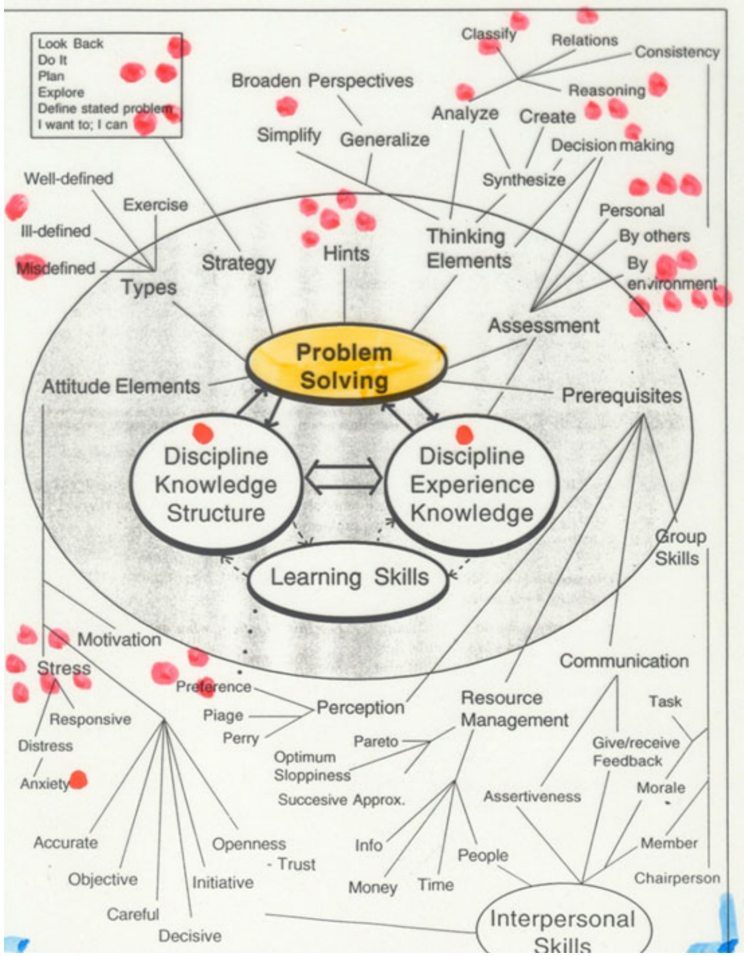
Build skill: activity + feedback + reflect

- Students gather own research evidence

Bridge skill: activity + feedback + reflect

- Post-test, Objectives
- DISCOVERY (more detailed reflection)

Include in reflective journal and **Extend** to everyday problems (i.e. Application of learning outside course)



Findings

The full McMaster program runs over multiple years (61 units in all) and the approach is incorporated in all Engineering units.

Works well for some units in isolation: awareness; strategy; self-assessment; analysis; creativity; stress management.

As a stand-alone course it contrasts strongly with other courses, which some students have trouble with. Overall feedback was positive. With some aspects of problem solving students showed improvement (awareness, use of strategy, creativity, reflection).

McMaster approach would work well if part of a broader program.

Major problem: not much time actually solving problems. Not much motivation for why we should do certain things. Reflection tasks should be more sparse.

2^{nd} iteration: why do we do the things we do, and how can we improve? (Alcock, Kahneman)

Addressing specific issues in mathematics:

- Students may be scared of mathematics.
- Students may try questions without knowing what all the words mean.
- Students might not have any problem solving strategy, or any techniques to try if they get stuck.

Approach: How to read and write mathematics, experience with definitions, theorems and proof (Alcock).

Approach: Uncover psychological challenges and barriers in problem solving and learn how to overcome them, e.g. biases, knowledge construction (Kahneman + MPS).

Approach: In-lecture lab work on mobile devices when engaged in algorithm development.

Run in blended mode, with large amount of online assessment and peer assessment.

Findings

Some success with the online assessment, and peer assessment.

Students had trouble with the more abstract self-reflective aspects of the course.

Mobile devices not a good teaching option without significant resource investment, too easy to be distracted. Frustration threshold very low.

Implications: discussing the psychology of learning and problem solving must be secondary to doing. i.e. the teacher deals with the psychology, is aware of the difficulties and addresses these invisibly when setting up the learning situation. Programming needs to be in the lab.

“all of the great educational theorists who have addressed mathematics education agree that learning is enhanced when students are given tasks which spark off activity in which familiar actions are adapted and modified in order to meet the challenge.” Mason *et al.*

3rd iteration: focus on solving problems (Zeitz)

"Toughen up, loosen up and practice" P. Zeitz

"Toughen up": Increase student frustration threshold by gradually increasing the number and difficulty of problems.

"Loosen up": provide tasks which require deliberate breaking of rules. Encourage shameless appropriation of new ideas. Allow students to play around, and fail (several failed attempts are fine provided the student keeps trying new approaches).

"Practice": provide lots and lots of problems. Solving them is not as important (it is healthy to have several unsolved problems hanging around).

An overview of the problem solving process: a mountaineering analogy and the heuristic approach (Zeitz)

1. **Strategy**: high level. E.g. climb easier surrounding peaks to observe target mountain from different angles.

Problem solving strategies for orientation phase: **“get hands dirty”, “make it easier”**

2. **Tactics**: middle level. E.g. if crossing a snowfield, go early in the morning.

Example problem solving tactics: **“draw a picture”, “factorise”**

3. **Tools**: lowest level. To cross snowfield, set up safety ropes, and use ice axes.

Example problem solving tools: **“completing the square”, “method of undetermined coefficients”**

Findings

Closest to method of Mason *et al.* Also used successful elements from earlier iterations.

Worked well in a small class (15 students). Keys to success are the nature of the problems, and the interaction with the teacher. Potentially scalable given appropriately scaffolded problems (more on this shortly).

Note: MPS was founded on the finding that doing lots of problems doesn't make students better problem solvers. But this should be amended: doing lots of problems works if done ***with reflection.***

Attitude…and what can the teacher do?

Most important aspect is attitude. Also often the most fragile part of the student as a problem solver.

Attitude-related goals for students:

- Must be free to make mistakes.
- Must be ready to push on when things don't immediately work.
- Must be able to self-check and correct.
- Ideally be comfortable in the class environment.
- Ideally be able to discuss with peers and provide feedback.

While there are interesting possibilities in relaxing this control, the teacher generally controls:

- The problems
- The activities
- The space
- The assessment

Building confidence, and "frustration threshold", with puzzles

Context-free nature makes students more inclined to engage, and less likely to bring along negative associations with any course material.

Directly exercises problem solving skills, and engages a student in mathematical thinking (problems not exercises)

Puzzles encourage reflective questions: what are we learning? How are we learning it? How are we using what we have learned?

System of equations:

1.1. Sue Ling is three times as old as Chin Lee was when Sue was as old as Chin is now. When Chin is as old as Sue is now, Sue will be 56. How old are Sue and Chin now?

Hidden assumptions:

1. You are in the downstairs lobby of a house. There are three switches, all in the off position. Upstairs, there is a room with a lightbulb that is turned off. One and only one of the three switches controls the bulb. You want to discover which switch controls the bulb, but you are only allowed to go upstairs once. How do you do it?

Developing student awareness of approach (and loosening up)

Many students appear to have almost no awareness of how they go about solving a problem.

TAPPS + monitoring + reflection

TAPPS (Talk-Aloud Pair Problem Solving): One person is the “Problem Solver” the other is the “Listener”. Problem Solver must articulate how they are approaching a problem. Listener mustn’t solve problem, but instead should follow Problem Solver and stop them if they don’t understand. Can incorporate more formal elements such as checking for “monitoring” statements (e.g. “where am I now?”).

Recording the dialogue for later examination is useful for reflection, and also useful for others to hear.

Feedback to the problem solver

Awareness

	problem						listener
Number of silent periods	0	1	2	3	4	5	>5
Number of checks, double checks	>5	5	4	3	2	1	0
Amount of writing/ charting	>5	5	4	3	2	1	0

More loosening up: ‘anything goes’ brainstorming

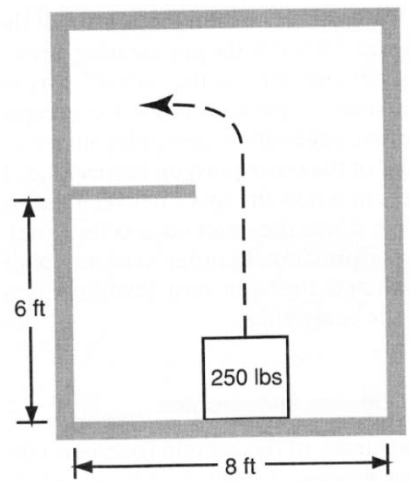
Brainstorming: All Ideas Good, Defer Judgement, Build on Others Ideas, Be Succinct: 50 Ideas in 5 minutes.

Goals: disabling the internal monitor, and building on others’ ideas.

Students enjoy the process.

A quick way to explore possible avenues forward for a complex problem.

Design a system for a 125 lb person to lift a 250 lb weight from the floor and place it on a shelf 6 ft high in a closet 8 ft wide by 8 ft deep.



Process worksheets: scaffolded problem-solving

Goal: provide some guidance to the student while they are approaching a problem (e.g. strategies), guidance on in-problem reflection, and what to do when stuck.

Certain things might be ticked as worth trying for a given problem.

Orientation

- ☐ Draw a picture
- ☐ Identify the knowns
- ☐ Identify the unknowns
- ☐ Identify the constraints
- ☐ Introduce notation

Strategy

- ☐ Make it easier
- ☐ Get your hands dirty
- ☐ The penultimate step
- ☐ Change point of view
- ☐ Work backwards

Tactics and Tools

- ☐ Factoring
- ☐ Symmetry
- ☐ Look at the extremes
- ☐ Crossover
- ☐ Add zero creatively

Looking back

- ☐ Check your result
- ☐ Can you see your result in a glance?

What was the crux move?

Building knowledge framework with diagrams

Concept mapping

Useful way to arrange knowledge and provides a quick indication to the instructor of how students are viewing different parts of a course.

Students surprisingly poor at mapping. Need to see examples.

Structure: Many possibilities, however one method is to look at concepts hierarchically. Nature of any connections should be identified. Examples can be included.

Introducing formal reflection activities

"The most talked about and least used of Polya's four stages" Mason *et al.*

Used sparingly: self-assessment towards specific targets, and more general "Discovery" sheets to reflect on learning. Used at end of course: a final reflection piece, citing evidence from activities within the course.

Evidence-based targets	Progress toward internalizing these targets				
	20%	40%	60%	80%	100%
Be skilled in describing aloud your thoughts as you solve problems					
Your problem solving skill improves if you pause and *reflect* about the process and about what you are doing					
You have your own particular style that works for you; others have a different preferred style					

Self-Assessment: DISCOVERY

Activities	Discovered	Application

Final Course Reflection (10%)

1. Attitude is perhaps the one biggest thing which can determine success or failure as a problem solver. Why might this be? What attitudinal characteristics do successful problem solvers have? Can you show evidence of these characteristics in your own problem solving? Cite the evidence, and the problem solving context in which it applies, in your response.

Working together: forming and using groups (Nilson, Zeitz)

Things to try:

- Have students write their own group contract and sign.
- Have students take a particular role in a group (finisher, organiser etc.)
- Make students aware of basic group dynamics (forming, storming, norming, performing), and types of individual behaviour which may be observed .
- Have students reflect on the group process.

Setting up groups:

- 3 or 4 per group works best
- People who enjoy each other’s company
- Fairly homogeneous maths skill/ability
- People of similar introvert/extrovert level (unless the dominant person is perceived to be one of the weaker students)
- People who usually, but not exclusively, play distinct roles (e.g. finisher, organiser, dreamy creative type, etc.)

Things to avoid: too many close friends together; imbalanced groups (such as brilliant extrovert with shy weaker students).

Assessed group work: Problem based learning in the lab

Open-ended problems, submitted as a group (lab report), may be mathematical (e.g. maximum overhang for a tower of concrete blocks, with no adhesive) cross-domain (e.g. design a launcher which launches a projectile a maximum horizontal distance before stopping, given specified materials and frictional parameters), or 'Fermi problems':

Problem Statement:

1. Design a launcher which launches a 2 kg projectile as far as possible horizontally given the materials and constraints detailed below.

This week we will be using some of the techniques of estimation discussed in the lectures during the week, supplemented by some model development using dimensional analysis, and one calculation question. This is to be completed in your groups.

1. Estimate the **total amount** of cooking oil used in Australia per day, and give this in units of per person per day.
2. Estimate the proportion of car travel in Australia which could be powered by this used cooking oil (once filtered, for use in diesel cars).
3. Use dimensional analysis to come up with an equation for the force on a moving car due to

Assessment marked by me

Labs, test and exam are all marked by me, using criteria to provide feedback on competency

Characteristic	Assessment of characteristics based on perf		
	Not demonstrated	Partially demonstrated	Demonstrated
Group members with roles specified			
Overview of problem solving process			
Problem breakdown provided			
“Best” solution task 1			
Justification for best, task 1			
Correctness of justification, task 1			
Adherence to constraints, task 1 and 2			

Category	Q. No.	Characteristic	Assessment of characteristics based on performance			
			Not proficient	Partially proficient	Proficient	Advanced
C course	1	Knowledge of TAPPS				
R course	1	Awareness of own problem solving approach				
C maths	2	How to approach mathematics (theory)				
R maths	2	Approaching mathematics (reflection on own process)				
C course	3	Characteristics of good problem solvers				
P course	4	Communication of approach				
P context-free	4	Context-free problem solving				
R course	5	Stress-level assessment				

Peer-assessment

Some assessment marked by peers, with a grade for the submission and a grade for the assessment.

All criteria-based, with many criteria, but simple quantitative judgement for which there can be little dispute (“How many ideas were generated?” “How many reflective questions were addressed?”), followed by feedback from the marker to the person who submitted the work.

Used “workshop” module on Moodle to manage the online submission process.

Findings (of how the assessment went)

All students have devices, so online submission was fairly straightforward (although our wireless was not up to managing 80 simultaneous devices).

Students didn't mind the peer-assessment, though many thought they could mark their own work better.

From the Learning Management Side (Moodle), technology is not really there yet. It was hard work to get Moodle to do what I want (or at least something satisfactory). Most things appeared "bare bones" (submit here, click there), and so not very engaging for students.

Even mature environments which are attempting to address the shortfalls (e.g. Piazza), while much better, are limited in what you can do. Next generation looks promising (Smart Sparrow, Open Learning etc.) but they don't yet have all the functionality of something like Moodle.

Group work went well, though I found it best to assign the groups. I also provided information on the group work process ("Forming", "Storming" etc.)

Online submission and peer-assessment saved significant resources in this course. Potentially useable for more complex tasks, but students need training and guidance in assessment.

Overall

Ideally course has a single discipline focus.

From McMaster: single target sheet referred back to many times would be useful. As would a single reflective “Discovery” sheet.

TAPPS, brainstorming and concept mapping are all useful, ideally used sparingly.

Puzzles provide a low threshold entry point.

Techniques such as dimensional analysis and “guesstimation” provide access to more open ended problems while practicing problem solving skills.

Group work is useful, but assessment is tricky.

Peer assessment is valued, but setting up rubrics and guidelines is tricky.

Ultimately useful to follow Zeitz’s maxim: “Toughen up, loosen up and practice”

The challenge of “actions for oneself” (self-initiated) vs “actions in onself” (require triggering) Vygotsky (via Mason)

Specific student challenges with the heuristics (e.g. how to make a problem simpler)

Resources

1. "Thinking mathematically", 2nd ed., J. Mason, L. Burton and K. Stacey
2. "How to think like a mathematician", K. Houston
3. "How to study for a mathematics degree", L. Alcock
4. "The art and craft of problem solving", 2nd ed., P. Zeitz
5. "Street-fighting mathematics", S. Mahajan
6. "Guesstimation 2.0: Solving today's problems on the back of a napkin", L. Weinstein
7. "How learning works", S.A. Ambrose *et al.*
8. "Teaching at its best", L.B. Nilson
9. "How to solve it", G. Polya
10. "Thinking, fast and slow", D. Kahneman
11. "Mathematics Teaching Practice: A Guide for University and College Lecturers", J.H. Mason
12. "Puzzle-based learning: An introduction to critical thinking, mathematics and problem solving", Z. Michalewicz and M. Michalewicz
13. McMaster Problem Solving course http://chemeng.mcmaster.ca/sites/default/files/media/mps2.pdf
14. "My Best Mathematical and Logic Puzzles", M. Gardner
15. "Problem Solving & Comprehension", A. Whimbey, J. Lochhead and R. Narode

(TAPPS is described in Ref. 15 and on the McMaster site (not the pdf above))