A Survey of Significant Developments in Undergraduate Mathematics Education Over the Past Decade

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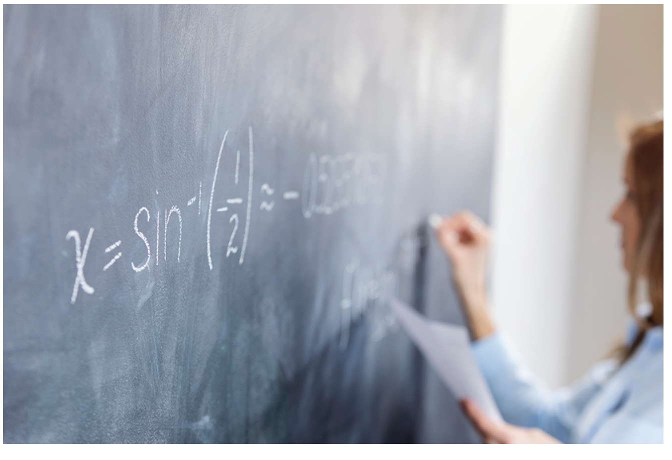

In this article I hope to provide some thoughtful responses to the organizing question **"What are the most significant results, events, or developments in undergraduate mathematics education of the last decade?"** To help answer this question, I posed it to approximately two dozen individuals prominent in the mathematics community and received timely responses from about two-thirds. For several years I have been one of the very few individuals in the federal government whose job description included the responsibility to "promote excellence in undergraduate mathematics education for all students in the United States."[1] In this capacity I have had a specialized perspective on the issues and concerns affecting undergraduate mathematics education and developed a particular network of contacts that I consulted to address the central question of this article. The individuals I surveyed included current and former program officers at the National Science Foundation; staff members (including past and present presidents), committee chairs, and officers of several national mathematics organizations; prominent researchers in undergraduate mathematics education; and other professors of mathematics. I recognize and acknowledge that the responses I received do not comprise a complete or representative sample of the possible answers to this question. There are topics that are over-represented in the subsequent discussion as

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[1]*The mission of NSF's Division of Undergraduate Education is "To promote excellence in undergraduate science, technology, engineering, and mathematics (STEM) education for all students."*

well as other topics that are missing or under-represented (e.g. teacher education, upper-division mathematics topics, and major-specific courses like real analysis and abstract algebra).

I have grouped the responses describing the most significant "results, events, or developments" in undergraduate mathematics over the last decade that I received into four distinct categories. My set of labels for these responses are **curricular pathways** (improving and modifying the trajectory students take to be prepared to be successful in college credit-bearing mathematics courses), **demographic changes** (addressing the underrepresentation of women and racial/ethnic minorities in all aspects of the mathematics community), **pedagogical innovations** (recognizing and incorporating active-learning and other evidence-based reforms in the teaching and learning of mathematics), and **expanding career options** (recognizing that the overwhelming majority of students who major in and take mathematics courses will not go on to become research mathematicians by providing all students with authentic experiences and numerous examples of how mathematics is actually used in non-academic settings).

CURRICULAR PATHWAYS

David Bressoud, DeWitt Wallace Professor of Mathematics, Macalester College, Minneapolis, MN; Director, Conference Board of the Mathematical Sciences (2017 to present); President, Mathematical Association of America (2009–2010).

I strongly suspect that the most far-reaching development in undergraduate mathematics of the past decade has gone on under the radar of most readers of the *Notices*. It is the emergence and widespread adoption of curricular changes typically known as "pathways projects." These arose in recognition of a serious problem: the low degree-completion rates of students who arrive at a two- or four-year undergraduate program with significant mathematical deficiencies. Traditionally, a student has had to pass a course of College Algebra before receiving any kind of degree. Large numbers of students, especially in two-year colleges but also in our four-year institutions, fail to qualify to start with College Algebra or a comparable course for which they can obtain college credit; instead they first enroll in a pre-college, non-credit-bearing course in mathematics. A study (Bailey *et al*, 2010) revealed that the prospects for these students have been abysmal, with only 15% eventually earning credit for College Algebra or another college credit-bearing mathematics course. The numbers of students involved are really quite large. In the fall term of 2015, 1.8 million students were enrolled in a pre-college, non-credit-bearing mathematics course, one million of them in four-year institutions, the others in two-year colleges. (To get a sense of the scale of this problem there are 550,000 students who took Calculus 1 at the post-secondary institution level and another 800,000 at the high school level in the 2015–16 academic year.) The problem is not just passing the remedial course or courses. Numerous studies have shown that even students who do well in their pre-college non-credit-bearing courses often fail to persist. Even with a 60% pass rate, which is typical, we often find 40% or more of those who *pass* these remedial courses decide not to continue their college education. Students who must pass two or more remedial courses before being allowed into a credit-bearing course are very unlikely to complete a degree. Compounding the problem is the fact that College Algebra, the terminal mathematics course for most of these students, often serves their future academic needs very poorly. The pathways programs, initiated by the Carnegie Foundation for the Advancement of Teaching and supported by the National Science Foundation[2] and soon followed by the Dana Center at the University of Texas, Austin, have been designed to ensure that all students entering with a need for remediation can enroll in a one-year course that carries college mathematics credit. Instead of College Algebra, the college-level course is usually either statistics or quantitative reasoning. The first semester is designed to prepare students for the credit-bearing course, focusing on the specific competencies, skills, and understandings that students will need for success in the second course. The results are outstanding, with pathways programs now reporting around 70% of these students successfully completing the sequence.

Jim Ham, President, American Mathematical Association of Two-Year Colleges (AMATYC), 2017–2018. Associate Professor of Mathematics, Delta College, University Center, MI.

The pathways initiative is the most significant trend in undergraduate mathematics education over the last decade. Its goal has been to streamline the developmental mathematics sequence, thereby reducing the time and credit hours necessary for students to attain proficiency in college level mathematics coursework. To achieve this, mathematics departments have created accelerated and corequisite curricular models, changed the initial placement procedures, and altered the developmental mathematics curriculum to align better with each pathway. To date, more than half of the mathematics departments at two-year colleges in the US have implemented pathways-recommended changes.

Jane D. Tanner, President, AMATYC, 2016–2017. Professor Emerita of Mathematics, Onondaga Community College, Syracuse, NY.

In my opinion the most significant development in undergraduate mathematics of the last decade is the realization that every student does not need to take college

[2] *DUE-1820830: "Scaling Up through Networked Improvement (SUNI): Testing a practical theory about improving math outcomes for developmental students at scale" and DUE-1322844: "Transforming Students' Mathematical Experiences: Advancing Quality Teaching with Reliability at Scale."*

algebra in order to be successful in their degree programs. The creation of multiple pathways has enabled non-STEM students to avoid the stumbling block of algebra and to complete their degree programs. Our students who are working on STEM degrees now have appropriate content that allows them to be better prepared for the calculus sequence. Increased collaboration among the professional organizations has allowed these pathways to be implemented at all levels of post-secondary mathematics education.

EXPANDING CAREER OPTIONS

Rachel Levy, Vice President for Education, SIAM, 2015–2018; Professor of Mathematics & Associate Dean for Faculty Development, Harvey Mudd College, Pomona, CA; Deputy Executive Director, Mathematical Association of America (MAA).

In the past decade there have been significant developments with respect to the ways that undergraduate mathematical sciences education can lead to BIG mathematics jobs (in business, industry, and government). Internships and computer science are both playing a larger role in the undergraduate experience and recruiters are becoming more aware of the value of a degree in the mathematical sciences. Sometimes programming is required for mathematical sciences majors, other times students elect supplementary courses, participate in activities such as hackathons, or learn to program as part of an additional major/minor. Classroom education is being enhanced by workplace and online learning experiences. Students are developing stronger communication skills, and bringing them to bear in mathematical presentations and work environments. Mathematical modeling is growing in K–12. However, the access to each of these opportunities is not yet well-distributed. This presents a challenge for us all.

Padmanabhan Seshaiyer, Associate Dean for Academic Affairs & Professor of Mathematical Sciences, George Mason University, Fairfax, VA; NSF Program Officer (Division of Mathematical Sciences), 2014–2016.

One of the significant paradigm shifts in undergraduate mathematics education in the last decade has been the willingness of faculty and students to engage in multidisciplinary research moving away from the traditional philosophy of "here is the mathematics, go solve the problem" to a more applied philosophy of "here is the problem, let us find the mathematics to do it." While the intellectual merit has continued to be on developing the fastest, cheapest, or the most accurate method for solving complex systems, there has been an increasing focus on the broader impact of the method to real-world applications. Some examples include probabilistic and stochastic approaches to solve the problem of poaching to the solution of systems of differential equations to study the spread of infectious diseases such as Zika to the use of machine learning in predictive data analytics. This has also helped major organizations to rethink their priorities such as the ten Big Ideas from the National Science Foundation, the fourteen Grand Challenges from the National Academy of Engineering, and the seventeen goals from the United Nations 2030 Sustainable Development Goals. To solve these global challenges there has been a significant focus in undergraduate mathematics education in training the next generation workforce on mathematical modeling, analysis, and simulation. Another major development has been new initiatives and programs to promote the much-needed awareness of celebrating [demographic] diversity in mathematical sciences through social media, movies, and conferences.

Suzanne Weekes, Professor of Mathematical Sciences, Worcester Polytechnic Institute, Worcester, MA; Chair, Committee on Education, SIAM.

I have seen a growing focus on getting students prepared for the non-academic workforce both at the undergraduate and graduate levels. There is certainly a demand in the job market for well-trained problem solvers who can already communicate effectively, who know how to learn new things, and are comfortable working on complex problems especially those involving data and those requiring solid quantitative skills. The way a lot of us are bringing things together for our students is by having our students work on real-world problems coming directly from industry. We have done this with our students at WPI through our senior research projects and in our Research Experiences for Undergraduates (REU) Program in Industrial Mathematics and Statistics for around two decades. Now, in the last five years, mathematics faculty all over the US have been able to participate in the NSF-funded PIC Math program[3] to help them prepare students at their own universities for industrial careers by engaging their students on research problems that come directly from industry. So far, PIC Math has worked with faculty from over 100 US institutions who have collectively engaged more than 1400 students on industrial mathematics research problems.

William Yslas Vélez, Professor Emeritus, Department of Mathematics, University of Arizona; NSF Program Director (Division of Mathematical Sciences), 1992–1993.

What do mathematics majors need to be able to compete in today's marketplace? To me it is clear that "one size fits all" is not the answer. Insisting on one program of study for all mathematics majors does not serve the needs of our students. In many departments the traditional course of study in linear and abstract algebra, real analysis, and complex variables needs modification to entice students into the mathematics major and to prepare them to meet the many exciting applications of mathematics. Students

[3]*DMS-1722275: "Preparation for Industrial Careers in Mathematical Sciences" and DMS- 1345499: "Preparing Mathematical Sciences Students for Business, Industry, and Government Careers (Pre-BIG)."*

majoring in X find that adding mathematics as another major makes X more competitive.

PEDAGOGICAL INNOVATIONS

Benjamin Braun, Associate Professor of Mathematics, University of Kentucky, Lexington, KY; founding editor, AMS *On Teaching and Learning Mathematics* blog.

Three recent developments stand out in my mind. First, the MAA's NSF-funded National Studies of College Calculus program[4] has identified characteristics of successful college calculus programs that provide critical information to faculty seeking to improve calculus instruction. Second, the research on inquiry-based learning in college mathematics by Sandra Laursen and her associates [in Ethnography & Evaluation Research at the University of Colorado Boulder] has solidly established the positive impact that active and inquiry-based learning has on college students. Third, the increasing level of collaboration among professional societies in mathematics over the past decade, most clearly demonstrated in the Common Vision report, has led to a greater degree of coherence in national-level initiatives and support for mathematics education.

Chris Rasmussen, Professor of Mathematics and Statistics, San Diego State University, San Diego, CA; co-editor, *International Journal on Research in Undergraduate Mathematics Education*.

One major shift in undergraduate mathematics education is the focus on active learning, both from the broader mathematical community and as a specific research focus from the Research in Undergraduate Mathematics (RUME) community. As we found out in the census survey that we did as part of the Progress through Calculus grant,[5] 44% of mathematics departments report that active learning is very important for the precalculus to calculus 2 sequence. I find this remarkable. A decade ago I suspect the percentage of departments reporting this would have been much, much less. Professional societies are stepping up and doing their part, NSF has stepped up and made this a focus, and the RUME community is right there with folks, investigating what this looks like in classrooms, the benefits for learners, the knowledge and dispositions instructors need to do this well, the design of curriculum to support active learning, etc.

Edray Goins, Professor of Mathematics, Pomona College, Claremont, CA; President, National Association of Mathematicians (NAM), 2015 to present.

One of the most significant events in undergraduate mathematics education in the last decade was the adoption of the Active Learning Statement by the CBMS Council in 2016. The Council of the Conference Board of Mathematical Sciences (CBMS) consists of nearly all the professional organizations in the mathematical sciences, such as the American Mathematical Society (AMS) and the National Association of Mathematicians (NAM). We realized that we needed to make a strong statement about the future of our profession. On July 15, 2016, the CBMS Council unanimously adopted a lengthy position statement on Active Learning. To quote from this 11-page document: "[W]e call on institutions of higher education, mathematics departments and the mathematics faculty, public policy-makers, and funding agencies to invest time and resources to ensure that effective active learning is incorporated into post-secondary mathematics classrooms."

John Haddock, Professor of Mathematics, University of Memphis, Memphis, TN; NSF Program Officer (Division of Undergraduate Education), 2004–2006, 2013–2017.

It is well recognized that Mathematical Sciences departments in the United States often rely heavily on graduate teaching assistants (GTAs) for teaching undergraduate students, and many of these same graduate students enter the workforce with teaching and mentoring undergraduates as an important component of their professional responsibilities. In line with this, there has been an increased interest by departments and professional organizations over the past few years to provide better preparation for GTAs in this realm (e.g. Transforming Post-Secondary Mathematics Education at `https://www.tpsemath.org` and College Mathematics Instructor Development Source at `cominds.maa.org`). At the same time, there have been several research efforts regarding how to train these GTAs, leading to the incorporation of education research findings such as understanding more about progressive stages of GTA teaching[6] and how to moderate them; development, vetting, and utilization of teaching resources[7]; and affirming that GTA training programs certainly can have positive impact in GTAs' students' successes, to cite just a few cases. For a variety of reasons, not the least of which is faculty fear that such programs will have negative effects on graduate student research, many departments have shied away from adopting these new GTA programs. Thus, of particular significance is the research finding that graduate preparation programs for teaching and mentoring undergraduates can be implemented without significantly increasing graduate students' time to graduation as shown in results from another NSF funded project.[8] In summary, much has been done, but it's just a drop in the bucket of what needs to

[4]*DRL-0910240: "Characteristics of Successful Programs in College Calculus."*

[5]*DUE-1430540: "Progress through Calculus."*

[6]*DUE-1744139: "EAGER: Exploring Mathematics Graduate Teaching Assistants' Developmental Stages for Teaching."*

[7]*DUE-1432381: "Improving the Preparation of Graduate Students to Teach Undergraduate Mathematics."*

[8]*DUE-0817537: "Impact of Professional Development Programs on Future STEM Faculty: A Mixed-Methods Longitudinal Study."*

be done to improve undergraduate student success in the mathematical sciences.

David Bressoud, DeWitt Wallace Professor of Mathematics, Macalester College, Minneapolis, MN; Director, Conference Board of the Mathematical Sciences (2017 to present); President, Mathematical Association of America (2009–2010).

In 2012, the President's Council of Advisors on Science and Technology (PCAST) castigated the mathematics community for instruction that was "dull and unimaginative," recommending that introductory courses be turned over to "faculty from mathematics-intensive disciplines other than mathematics." Later that year, the National Research Council published its report on the state of Discipline-Based Educational Research. Efforts by the mathematics community were notably absent. It was not the case the mathematics community had been doing nothing to improve instruction, but its efforts were not particularly visible, especially at the research universities that prepare the majority of our science and engineering majors. Research universities are increasingly aware of this need. In 2015, over 40% of PhD-granting departments of mathematics acknowledged the use of active learning practices as "very important"; 35% of PhD-granting departments were at least experimenting with the use of active learning practices in some sections of calculus.

DEMOGRAPHIC CHANGES

Linda Braddy, Vice President for Academic Affairs, Tarrant County College; Deputy Executive Director, Mathematical Association of America, 2012–2016.

The increasing awareness of issues related to diversity and inclusion. More and more research is being done on the ways in which our traditional methods of teaching and interacting with students hinder their success in mathematics, particularly students from underrepresented populations. We are far from where we need to be with regards to the use of inclusive pedagogies in our classrooms. Progress is predicated on our willingness to acknowledge and ability to overcome our implicit biases and mistaken notions about student ability and to instead base our perceptions and actions on evidence from current research in the field.

Chris Rasmussen, Professor of Mathematics and Statistics, San Diego State University, San Diego, CA; co-editor, ***International Journal on Research in Undergraduate Mathematics Education*****.**

Another more recent shift in undergraduate mathematics education is a focus on equity, diversity, and inclusive practices. A few years ago, you would not have found many talks on this subject at the annual Research in Undergraduate Mathematics (RUME) conference, but nowadays these are prominent and very well attended. Consistent with the first theme that is going beyond individual accounts (which often take on a deficit stance), research is examining the gendered and racialized practices across levels (classrooms, departments, institutions, society). I feel very optimistic that this new emphasis in RUME will continue and do some good. The most recent Inquiry-Based Learning (IBL) conference in summer 2018, which was previously associated with the legacy of R. L. Moore, was abound with excellent talks on equity. This is a good sign.

I end this article with a few caveats and observations. First, the responses included here reflect the views of those who submitted them and do not reflect the views of my employers (National Science Foundation or Occidental College). Second, these responses are obviously not from a randomly selected, representative sample of the mathematics community, so there really is no question of whether they span the basis of the entirety of mathematics opinion. Third, the inclusion or exclusion of particular projects or programs does not constitute an endorsement or disparagement by the author. Overall, I believe that the responses included above, taken as a whole, provide some insight into the most important developments in undergraduate mathematics education over the past decade. I hope that readers will use this information to promote excellence in undergraduate mathematics for all students in the next decade and beyond.

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