A COMMON VISION

for Undergraduate Mathematical Sciences Programs in 2025

Karen Saxe

Linda Braddy

Foreword by William "Brit" Kirwan

MATHEMATICAL ASSOCIATION OF AMERICA

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A Common Vision

for Undergraduate Mathematical Sciences Programs in 2025

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About the project

The *Common Vision* project is a joint effort, focused on modernizing undergraduate programs in the mathematical sciences, of the American Mathematical Association of Two-Year Colleges (AMATYC), the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM).

Acknowledgements

Thanks to funding from the National Science Foundation (NSF DUE-1446000), we were able to bring together individuals with extensive experience related to undergraduate curricula in the mathematical sciences to offer guidance on this project.

This report represents the collective wisdom of many individuals, and we would like to express our gratitude to all who participated. This project included a two-and-a-half-day workshop held in May 2015 at ASA headquarters in the Washington, D.C. area. Participants (listed in Appendix A) represented all of the professional associations that have as one of their primary objectives the increase or diffusion of knowledge in one or more of the mathematical sciences, as well partner disciplines in science, technology, and engineering. We do not view the distinct efforts of various associations as competing efforts, but instead as the basis and strong foundation for collective action that is well-informed by a variety of perspectives. We were fortunate to also engage participants from outside the academy, from higher education advocacy organizations, and from industry. Much of the work for the forward-looking portions of this report was done at the workshop, and we appreciate the expertise and enthusiasm of the workshop participants who did this work and also provided feedback on subsequent drafts of the report. When working within such a diverse group, there can be communication challenges; e.g., “pathways” can mean different things depending on institutional context. We are especially grateful that everyone was so willing to approach the work with an open mind and a sense of humor.

The leadership team members (listed below) contributed significantly to planning the project, writing the first draft, running the workshop, serving on panels at conferences, and generally providing information and great wisdom to the authors. It has been a true joy to work with this group. Finally, we are grateful to ASA and MAA staff for their critical support on everything from setting up the *Common Vision* website to handling the logistics of the May 2015 workshop.

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Foreword

As one long involved in higher education, and in particular, education in the mathematical sciences, I understand the current challenges we face in STEM education across the board. Major changes in the field of mathematics itself, expanding opportunities for collaboration with other STEM and non-STEM disciplines, growing economic pressures, and rapidly changing technologies throughout higher education have triggered unprecedented national focus on mathematical sciences education. Two influential reports clearly articulated many of the specific challenges we face:

(1) The President's Council of Advisors on Science and Technology's (PCAST) 2012 *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics* reported dissatisfaction in how undergraduate mathematics is taught to students outside the mathematics major. Further, outdated course materials and teaching techniques have not provided students with the quantitative skills demanded for employment and good citizenship.

(2) The National Research Council's (NRC) 2013 *The Mathematical Sciences in 2025* called for mathematics teaching that better aligns with other disciplines.

I currently serve as senior advisor to Transforming Post-Secondary Education in Mathematics (TPSE Math) which aims to effect constructive change in mathematics education. As our vision statement puts it, we believe that post-secondary education should "enable any student, regardless of his or her chosen program of study, to develop the mathematical knowledge and skills necessary for productive engagement in society and in the workplace." As a member of the TPSE leadership group, Karen Saxe, one of the authors of this report and leaders of the Mathematical Association of America's (MAA) *Common Vision* project, clearly shares this vision.

What is truly extraordinary about *Common Vision* is that it brought together faculty and other leaders from all types of post-secondary institutions, from two-year colleges to research-intensive universities, representing the five professional associations whose missions focus on undergraduate mathematical sciences education to some extent. As chair-elect of the Executive Committee of the Conference Board of the Mathematical Sciences, I am heartened to see these five associations working together in such a substantive way. Each of the five has published curricular guides, and some were written without intense collaboration with the other associations. *Common Vision* examined these guides for the purpose of identifying common themes and articulating a coherent vision of undergraduate programs in the mathematical sciences that all five associations would endorse and promulgate. The ultimate goal of *Common Vision* was to galvanize the mathematical sciences community around a modern vision for undergraduate programs and to spur grassroots efforts within the community as a foundation for addressing the collective challenges we face and for capitalizing on the opportunities outlined in the NRC and PCAST reports.

I appreciate the MAA's leadership in securing funding for *Common Vision* as well as the knowledge and expertise that authors Karen Saxe and Linda Braddy, Deputy Executive Director of the MAA, contributed to it. I am pleased to see this *Common Vision* report come to fruition, as it provides a snapshot of our community's view of what should be taught in collegiate mathematics, how it should be taught, and how we can move forward and implement the changes necessary to equip our students with the knowledge and skills necessary to meet the demands of the 21st century.

TPSE Math and *Common Vision* were both launched to address similar challenges and capitalize on the power of collective action, and I am pleased to see such coalitions forming to address the myriad challenges facing the mathematical sciences community.

William "Brit" Kirwan
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Executive Summary

The *Common Vision* project brought together leaders from five professional associations — the American Mathematical Association of Two-Year Colleges (AMATYC), the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) — to collectively reconsider undergraduate curricula and ways to improve education in the mathematical sciences. Project participants represented not only these mathematical sciences associations, but also partner STEM disciplines, higher education advocacy organizations, and industry.

We began with an in-depth examination of seven curricular guides published by these five associations and spent a substantial amount of time identifying common themes in the guides. This report reflects a synthesis of these themes with our own research and input from project participants and other thought leaders in our community.

One of the most striking findings is that all seven guides emphasized this point, in particular:

The status quo is unacceptable.

Consequently, this report focuses on specific areas that require significant further action from the mathematical sciences community to improve undergraduate learning, especially in courses typically taken in the first two years. These areas fall into one of four categories: curricula, course structure, workforce preparation, and faculty development.

In this report, we call on the community to (1) update curricula, (2) articulate clear pathways between curricula driven by changes at the K–12 level and the first courses students take in college, (3) scale up the use of evidence-based pedagogical methods, (4) find ways to remove barriers facing students at critical transition points (e.g., placement, transfer) and (5) establish stronger connections with other disciplines. Institutions should provide faculty with training, resources, and rewards for their efforts to adapt curricula, develop new courses, and incorporate pedagogical tools and technology to enhance student learning. Departments should update curricula, establish multiple pathways into and through majors, and move toward environments that incorporate multiple pedagogical approaches throughout a program. Instructors should present key ideas and concepts from a variety of perspectives, employ a broad range of examples and applications to motivate and illustrate the material, promote awareness of connections to other subjects, and introduce contemporary topics and applications. Students should learn to communicate complex ideas in ways understandable to collaborators, clients, employers, and other audiences.

To ensure students graduate with skill sets to match expectations of prospective employers, our community must modernize curricula with input from representatives in partner disciplines, business, industry, and government. This work should aim to narrow the gap between mathematics as practiced in the academy and other employment sectors and mathematics as experienced in higher education's instructional programs. While intellectual domains fragment and coalesce over time, a central task for mathematics faculty at institutions of higher education, and more broadly, the mathematical sciences community as a whole, is to create a coherent, intriguing introduction to collegiate mathematics for all students.

Moving forward, we believe it is critical to maintain the collective connection and dialogue among the associations established during this initial phase of *Common Vision*. This phase sought to effect changes in undergraduate mathematical sciences education in order to expand scientific knowledge and maintain a viable workforce in the United States. By reaching out to members of the five associations, we hoped to galvanize the mathematical sciences community and spur grassroots efforts to improve undergraduate education. Change is unquestionably coming to lower-division mathematics and statistics, and it is incumbent on the mathematical sciences community to ensure it is at the center of these changes, not on the periphery. We hope other individuals and groups will come alongside us in this effort, capitalize on the momentum we have built and goodwill we have established, and move this effort forward into a second phase focused on implementation initiatives.

1 Introduction

Freshman and sophomore mathematics and statistics courses function as gateways to many majors, and they are crucial for preparing mathematically- and scientifically-literate citizens. Yet:

- Each year only about 50 percent of students earn a grade of A, B, or C in college algebra (Ganter & Haver, 2011).
- Women are almost twice as likely as men to choose not to continue beyond Calculus I, even when Calculus II is a requirement for their intended major (Bressoud, 2011).
- In 2012, 19.9 percent of all bachelor's degrees were awarded to underrepresented minority students (9.5 percent to Blacks, 9.8 percent to Hispanics). However, only 11.6 percent of mathematics bachelor's degrees were awarded to underrepresented minority students (4.9 percent to Blacks, 6.4 percent to Hispanics) (www.nsf.gov/statistics/nsf07308/content.cfm?pub_id=3633&id=2).
- Failure rates under traditional lecture are 55 percent higher than the rates observed under more active approaches to instruction (Freeman et al., 2014).

Additional challenges are outlined in reports such as *The Mathematical Sciences in 2025* (National Research Council [NRC], 2013) and *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics* (President's Council of Advisors on Science and Technology [PCAST], 2012). These reports have led to varied responses from subgroups within the mathematical sciences, including the launch of this *Common Vision* project. It is time for collective action to coordinate existing and future efforts in such a way that the mathematical sciences community is pulling in the same general direction and leveraging the collective power of the whole to improve student success, especially in the first two years of college. This project is intended as a new beginning, marking a period of increased interaction and collaboration among all stakeholders to improve post-secondary education in the mathematical sciences.

Over the past several decades, members of various mathematical sciences professional associations have devoted much thought to educational issues and published distinct curricular guides. This *Common Vision* report integrates them into a single set of recommendations and provides a snapshot of the current collective thinking about undergraduate education in mathematics and statistics. It lays a foundation for future work that acknowledges the changing face of the mathematical sciences, particularly with respect to the inclusion of data science, modeling, and computation.

Undergraduate courses in the mathematical sciences that students take during the first two years range from developmental courses to regression analysis and differential equations; our focus is the collection of credit-bearing courses (i.e., "non-developmental") in the mathematical sciences typically taken in the first two years. In this report, we examine the undergraduate program, including statistics, modeling, and computational mathematics as well as applications in the broader mathematically-based sciences using a wide-angle lens. We include actuarial studies and operations research, engineering and the physical sciences, the life and social sciences, and quantitative business topics like accounting. We examine the issue of multiple "pathways" in a variety of contexts, with our use of this term intended to encompass (1) pathways into majors in the mathematical sciences, (2) pathways through these majors, and (3) pathways through general education mathematics and statistics requirements. We include developmental curricula only when it cannot be divorced from general education issues. We recognize the importance of K–12 and developmental curricula in the full elementary-through-baccalaureate spectrum, but including recommendations such as those from the National Council of Teachers of Mathematics (NCTM) is beyond the scope of this project.

The *Common Vision* project brought together leaders from five professional associations — the American Mathematical Association of Two-Year Colleges (AMATYC), the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) — to collectively reconsider undergraduate mathematics curricula and ways to improve education in the mathematical sciences at a two-and-a-half day workshop in May 2015 at ASA headquarters in the Washington, D.C., area. Workshop attendees represented not only the five aforementioned mathematical sciences associations, but also partner STEM disciplines, higher education advocacy organizations, and industries.

In this major cooperative effort to improve teaching and learning, we joined forces to address a wide range of issues affecting the critical first two years of collegiate mathematics and statistics. *Common Vision* is grounded in an in-depth examination of seven curricular guides published by the five professional associations involved in this project. We spent substantial time identifying common themes in these guides and determining areas for future work to improve undergraduate learning in the mathematical sciences, especially in courses typically taken in the first two years. One of the most striking findings is that all seven guides emphasized this point, in particular:

The status quo is unacceptable.

Essential to bringing about lasting change is a continuation of this collective *Common Vision* effort. We do not suggest a single prescription for all contexts but, rather, that work be done in counterpoint to create a tapestry of ideas.

We envision this project as Phase I of a two-part initiative. Phase I focuses on introspection: we seek internal coherence of vision within the mathematical sciences community. Phase II of the project will be an outward-looking period focused on widespread dissemination and large-scale implementation of modernized curricula and delivery methods. We are aiming to narrow the gap between today's mathematics as it is practiced in the academy, industry, and government and how it is experienced in higher education's instructional programs. Part of this modernization is recognizing the essential nature and breadth of mathematics as manifested in its extraordinary power to advance other fields and to find within these fields new problems worthy of mathematicians' scrutiny. While intellectual domains fragment and coalesce over time, we believe that a central task for mathematics faculty at institutions of higher education, and more broadly, the mathematical sciences community as a whole, is to create a coherent, intriguing introduction to collegiate mathematics for all students. As we move forward, it is critical that we maintain the collective connection and dialogue among the associations we have established during this initial phase. We must also build on work done at higher education institutions that merits emulation and further study.

There is a great deal of follow-up work to be done investigating the efficacy of existing recommendations related to undergraduate curricula and instructional methodologies. Evidence of effective programs exists, and many recommendations deserve additional consideration. Which recommendations have been implemented, and which have resulted in improved student outcomes? This project builds on past and current work as a way to avoid repeating our mistakes, re-inventing successful initiatives, and pursuing misguided directions. A central goal of the project is to motivate efforts to modernize programs in the mathematical sciences and produce an adequate number of graduates with strong mathematical competencies for the United States workforce. We examine past work and identify promising practices in curricula revision for the purpose of making exemplary curricular and pedagogical initiatives visible to the broader community as the impetus for widespread implementation of modernized courses. This report can also guide funding agencies regarding best bets for targeted federal investments likely to have transformative impact within the community.

As practitioners in the mathematical sciences, we celebrate the beauty and power of mathematics. Courses in the mathematical sciences have been taught as part of a classical education for thousands of years and continue to gain new meaning and relevance. There are now, perhaps more than ever, amazing career opportunities for people with training in mathematically-intensive fields. Rapid advances in technology and in connections between mathematics and other fields present tremendous opportunities, and the mathematical sciences community is at a pivotal point. Politicians across the country and mathematical scientists, not just mathematics educators, are more keenly focused on undergraduate mathematics and statistics education issues than in the past. We are attempting to capitalize on the current climate to (1) advance our goal of a shared vision for modernized curricula and pedagogies and (2) improve the public perception of our field.

The mathematical sciences are in the national spotlight in part because mathematical competencies can lead to higher paying jobs, and thus can play a profound role in students' economic mobility. There is substantial support for reforming undergraduate instruction in the mathematical sciences from influential actors: the White House Office of Science and Technology Policy, the National Academies, and the Association of American Universities, among others. We have taken advantage of this increased attention at the national level by writing a white paper for Dr. John Holdren, who is Assistant to the President for Science and Technology, Director of the White House Office of Science and Technology Policy, and co-chair of PCAST. We sent the paper to several Congressional members as well, including chairs and ranking members of the House and Senate committees on education. We also wrote calls-to-action to members of the mathematical sciences community and opinion pieces aiming to raise the visibility of this project, e.g., in an AMS blog (Saxe, 2015, May) and in the Association for Women in Mathematics Newsletter (Saxe, 2015, July-August). In our view, public perception will change more rapidly if the mathematical sciences community speaks with a unified voice; to this end, we are very pleased that Conference Board of the Mathematical Sciences (CBMS) has endorsed the *Common Vision* project as part of the constructive response to reports critical of our community (NRC, 2013; PCAST, 2012). All these communiquès are available at the *Common Vision* website (www.maa.org/common-vision).

This report is organized as follows:

In Chapter 1, we include background for this project, a brief description of the NRC (2013) and PCAST (2012) reports, and profiles of the five professional associations.

In Chapter 2, we enumerate the common themes identified in the seven curricular guides.

In Chapter 3, we describe "next step" proposals developed by *Common Vision* participants for initiatives to advance this work.

1.1 Background

Our premises are:

- Mathematical scientists — including theoretical and applied mathematicians, statisticians, computational scientists, and mathematical sciences education researchers — contribute in fundamental ways to initiatives to advance national priorities in the interests of all citizens.
- The most productive approach to preparing the next generation of citizens literate in science, technology, engineering, and mathematics (STEM) will involve multidisciplinary teams of mathematical scientists, other domain specialists from STEM and non-STEM fields, and employers working together to modernize undergraduate mathematics and statistics programs.
- Mathematical sciences courses in the first two years of college function as pathways into many different STEM majors and also as key components in the preparation of scientifically-literate citizens.

Society benefits from college graduates who are generally educated in higher mathematics, whose lives and social activities are influenced by their understanding of mathematics and, through it, of interesting aspects of history and culture. Beyond career and employment issues, "pure" mathematics majors are parents, aunts, uncles, volunteers in schools, tutors, voters in elections, school board members. Pure mathematics courses, including those driven mainly by aesthetic concerns, can help prepare students to become valuable citizens, all of whose contributions are augmented by skills and habits of mind developed through learning mathematics (MAA, 2015, p. 61).

Specific ways in which knowledge in the mathematical sciences can contribute to societal problems are outlined in, for example, *The Future Postponed*, a report by the MIT Committee to Evaluate the Innovation Deficit (MIT, 2015).

1.2 Impetus to change

We initiated this project in response to national calls to improve undergraduate education in the mathematical sciences. These calls include, but are not limited to: *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics* (PCAST, 2012) and *The Mathematical Sciences in 2025* (NRC, 2013). These two reports in particular criticized the collective enterprise of teaching mathematics to undergraduates.

We are also responding to the fact that the higher education environment has undergone and continues to experience significant changes. Changes are particularly profound in the areas of:

- Student preparedness.
- Student diversity.
- Student career goals and the need for workplace skills (e.g., technology skills, data analysis skills).
- Quantitative skills demanded by more disciplines (e.g., the social sciences).
- Advances in technology (e.g., software for teaching, learning, and assessment; massive open online courses [MOOCs]; growth in the use of computation as a means for enhancing conceptual understanding).
- State budget cuts for post-secondary education and shifts in states' priorities from funding based on enrollment to funding based on completion.

In *The Mathematical Sciences in 2025*, NRC called for mathematics departments to rethink the types of students they attract and to identify the top priorities for educating these students. Change is unquestionably coming to lower-division undergraduate mathematics, and it is incumbent on the mathematical sciences community to ensure it is at the center of these changes, not on the periphery.

In their *Engage to Excel* report, PCAST acknowledged that fewer than 40 percent of students who enter college intending to major in a STEM field actually go on to complete such a degree. They concluded that retaining more STEM majors is the best option for addressing the inadequate supply of STEM professionals in the United States workforce. An additional aspect of the retention challenge is retaining underrepresented groups (minorities and women) in mathematical sciences degree programs at all levels, undergraduate through doctoral.

Our community recognizes that many students encounter significant barriers along the traditional route to a STEM career in their mathematics courses and thus possess inadequate mathematical competencies when they enter the workforce. *The Mathematical Sciences* *in 2025* (NRC, 2012) suggested that we re-assess the training of future generations of mathematical scientists in light of the increasingly cross-disciplinary nature of STEM fields. In its response to the PCAST report, the American Mathematical Society (AMS) affirmed the mathematical community's commitment to preparing students for success in STEM careers and asserted that it is essential for mathematicians to engage in designing and teaching courses that form the foundation of STEM education. It also referenced many current efforts in undergraduate education designed to address the challenges of teaching entry-level college mathematics. Further, it called for the mathematical sciences community to develop and share new approaches and teaching methods to enhance the learning experience for STEM majors (Friedlander et al., n.d.). Promising curricular updates and pedagogical practices have since been recommended. However, few such practices are being implemented at a scale necessary to substantially increase the number of mathematics graduates entering the workforce, the number of students pursuing a degree in the mathematical sciences, or the number of graduates in all fields who have adequate mathematics skills and competencies to meet current workforce demands. By bringing together thought leaders from various sectors within higher education and beyond, *Common Vision* will ultimately serve to catalyze widespread adoption of modernized curricula and pedagogies toward the goal of producing a more mathematically-literate citizenry.

1.3 The collective enterprise of teaching

Research on "collective impact" (Kania & Kramer, 2011) suggests that, in achieving significant and lasting change in any area, a coordinated effort supported by major players from all existing sectors is more effective than an array of new initiatives and organizations. The *Common Vision* project encourages such action by highlighting existing efforts and draws on the collective wisdom of a diverse group of stakeholders to articulate a shared vision for modernizing the undergraduate mathematics program.

It is thus critical to the success of the next phase of this project that we have participation from a broad range of professional associations. The Conference Board of the Mathematical Sciences (CBMS) is an umbrella organization consisting of seventeen professional associations, all of which have as one of their primary objectives the increase or diffusion of knowledge in one or more of the mathematical sciences. Five of the seventeen — the American Mathematical Association of Two-Year Colleges (AMATYC), the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) — focus on undergraduate education to some degree. Project leaders have thus been drawn from AMATYC, AMS, ASA, MAA, and SIAM.

AMATYC, founded in 1974, is the only organization exclusively devoted to providing a national forum for the improvement of mathematics instruction in the first two years of college. Central to its mission are promoting and increasing awareness of the role of two-year colleges in mathematics education, and communicating the perspectives of two-year college mathematics educators to public, business, and professional sectors. AMATYC has approximately 1,800 individual members and more than 150 institutional members in the United States and Canada.

AMS was founded in 1888 to further the interests of mathematical research and scholarship. Through its publications, meetings, advocacy, and other programs, the AMS supports mathematics education at all levels and fosters an awareness and appreciation of mathematics and its connections to other disciplines and everyday life. Members include almost 30,000 individuals and 580 institutional members worldwide.

ASA is the world's largest community of statisticians, the "Big Tent for Statistics." ASA supports excellence in the development, application, and dissemination of statistical science through meetings, publications, membership services, education, accreditation, and advocacy. Founded in 1839, ASA is the second-oldest continuously operating professional association in the country. Since its inception, the association has had a close affiliation with the statistical work of the United States government, particularly the Bureau of the Census. Today, ASA serves 18,000 members worldwide.

MAA is the largest professional association that focuses on mathematics accessible at the undergraduate level. Members include university, college, and high school teachers; graduate and undergraduate students; pure and applied mathematicians; computer scientists; statisticians; and many others in academia, government, business, and industry. The mission of MAA is to advance the mathematical sciences, especially at the collegiate level. MAA was established in 1915 and currently serves 17,000 individual and institutional members worldwide.

SIAM, incorporated in 1952, is an international community of over 13,000 individual members. Almost 500 academic, manufacturing, research and development, service and consulting, government, and military organizations worldwide are institutional members. Through publications, research, and community, SIAM pursues its mission to build cooperation between mathematics and the worlds of science and technology. SIAM's goals are to advance the application of mathematics and computational science to engineering, industry, science, and society; promote research that will lead to effective new mathematical and computational methods and techniques for science, engineering, industry, and society; and provide media for the exchange of information and ideas among mathematicians, engineers, and scientists.

2 Existing Recommendations

Phase I of our two-part initiative centers on introspection. Many publications have guided our thinking, and we have chosen seven curricular guides on which to focus. Each of these seven guides targets undergraduate courses and programs and is endorsed by the supporting association. At a time of rapidly declining membership numbers for many professional associations, it is relevant to reflect on the value of such organizations; we recognize the significant ongoing work of these associations' members assessing current practices and publishing recommendations for curricula and pedagogy.

- *Beyond Crossroads*, released in 2006, is AMATYC's update of the 1995 publication *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*. http://beyondcrossroads.matyc.org/
- *Guidelines for Assessment and Instruction in Statistics Education College Report* is ASA's 2005 publication with recommendations for introductory statistics curricula. ASA's Guidelines for Assessment and Instruction in Statistics Education (GAISE) project consisted of two groups focused on K–12 education and introductory college courses, respectively. This publication presented the recommendations developed by the college-focused group. http://www.amstat.org/education/gaise/
- *Curriculum Guidelines for Undergraduate Programs in Statistical Science*. These guidelines were endorsed by the ASA board of directors in 2014. http://www.amstat.org/education/curriculumguidelines.cfm
- The *2015 CUPM Guide to Majors in the Mathematical Sciences*. MAA's Committee on the Undergraduate Program in Mathematics (CUPM) is charged with making recommendations to guide mathematics departments in designing curricula for their undergraduate students. CUPM began issuing guidelines in 1953, updating them at roughly ten-year intervals. The most recent guide was published 2015. http://www.maa.org/sites/default/files/pdf/CUPM/pdf/CUPMguide_print.pdf
- *Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra*. Curriculum Renewal Across the First Two Years (CRAFTY) is a subcommittee of CUPM and is charged with monitoring ongoing developments in curricula for the first two years of college mathematics and making general recommendations. This guide was completed in 2011. http://www.maa.org/sites/default/files/pdf/CUPM/crafty/introreport.pdf

- *Modeling across the Curriculum*. The first SIAM guide provided a summary of the Modeling across the Curriculum workshop held in 2012 and makes recommendations on curricula in areas relevant to applied and computational mathematics. The second such workshop was held in January 2014, and investigated ways to increase mathematical modeling across undergraduate curricula and to develop modeling content for the K–12 educational arena. http://www.siam.org/reports
- *Undergraduate Programs in Applied Mathematics* was released by SIAM in 2014. http://www.siam.org/reports/undergraduate_14.pdf

Additional background on each of these guides is included in Appendix B. A number of other reports published within our community, which are listed in Appendix C, re-affirmed these recommendations and provided additional guidance for this report.

Topics discussed in the following section appear in all seven of the guides. In the subsequent section, we include other issues that appear in at least one of the guides or elsewhere in the literature that we believe warrant significant attention from the mathematical sciences community.

2.1 Common themes

We want to re-iterate the fact that all seven curricular guides declared that ***the status quo is unacceptable***. The specific areas that all the guides agreed require significant further action fall into one of four categories: curricula, course structure, workforce preparation, and faculty development. These are, of course, interdependent and do not exist in isolation; we acknowledge that any particular issue might fit in more than one of these categories. We also acknowledge that some of these are not under the exclusive control of faculty members and, thus, change will require the participation of others. Improving teaching and learning requires well-coordinated efforts by multiple stakeholders, including faculty, administrators, employers, professional associations, and funding agencies.

2.1.1 Curricula

While this project focuses on courses taken in the first two years of college, it is impossible to discuss them without an eye toward subsequent courses and the understanding that “an appropriate developmental progression is required to obtain mastery” (ASA, 2014, p. 9). While some of the guides recommended specific courses for the first two years, others did not, and hence we do not list course recommendations here.

All the guides agreed that instructors should present key ideas and concepts from a variety of perspectives, employ a broad range of examples and applications to motivate and illustrate the material, promote awareness of connections to other subjects, and introduce contemporary topics and their applications. Instructors should intentionally plan curricula to:

- Enhance students’ perceptions of the beauty, vitality, and power of the mathematical sciences.
- Enhance students’ understanding of mathematics as a creative endeavor.

- Increase students' quantitative and logical reasoning abilities needed for informed citizenship and for the workplace.
- Increase students' confidence and joy in doing mathematics and statistics.
- Improve students' ability to communicate quantitative ideas orally and in writing (and since a precursor to communication is understanding, improve students' ability to interpret information, organize material, and reflect on results).
- Encourage students to continue taking courses in the mathematical sciences.

More pathways. The mathematical sciences community must begin to think in terms of a broader range of entry-level courses and pathways into and through curricula for all students, including mathematics and other STEM majors as well as non-STEM majors. High national unemployment generates additional mathematics majors at higher education institutions (Bressoud, 2010), but we must remain vigilant during times of increased economic prosperity to ensure we continue to attract majors. Our community must remain attentive and design curricula to "address the needs of as many academic paths and disciplines as possible" (AMATYC, 2006, p. 38).

All seven of the guides call for multiple pathways into and through mathematical sciences curricula, some of which should include early exposure to statistics, modeling, and computation. Data-driven science is reshaping the processes of discovery and learning in the 21st century. The current attention to big data and the demand for college graduates with data skills should prompt changes in our entry-level courses which result in students being better prepared for jobs requiring computational and statistical skills. Thus, there is a call to provide mathematically substantive options for students who are not headed to calculus. These entry courses should focus on problem solving, modeling, statistics, and applications. Current college algebra courses serve two distinct student populations: (1) the overwhelming majority for whom it is a terminal course in mathematics, and (2) the relatively small minority for whom it is a gateway to further mathematics. Neither group is well-served by the traditional version of the college algebra course. There is a mismatch between a curriculum designed to prepare students for calculus and the reality that only a small proportion of these students subsequently enroll in calculus (MAA, 2012, p. 49). We acknowledge the need to focus on the calculus sequence and ensure that pathways to it remain a high priority, as calculus is central to most further study in the mathematical sciences, but it behooves us to develop curricula effective for the majority of the population as well.

Mismatched curricula and lack of satisfactory communication between mathematics departments and other academic units are not new. Indeed, according to the MAA (2004):

> … many view the formal study of mathematics as irrelevant or tangential to the needs of today's society. They see mathematics departments as disconnected from other disciplines except through a service component that they believe is accepted only reluctantly and executed without inspiration or effectiveness. Such views were expressed by a majority of the academic deans at the research universities sampled for the American Mathematical Society (AMS) study *Towards Excellence*: "The prevalent theme in every discussion [with deans] was the insularity of mathemat-

> ics. Mathematicians do not interact with other departments or with faculty outside mathematics, many deans claimed. The deans … seemed to view mathematics departments as excessively inward looking." This perception is often due more to poor communication than to a lack of effort or good intention. At the least, it points to the need for better communication (p. 3).

On a positive note, this is not the case for those of us who enjoy very good relations with partner discipline departments at our institutions. The AMS study cited by MAA was conducted in the 1990s, and we hope our more recent personal experiences signal that some progress has been made on this front.

There are existing examples of various pathways into mathematics that include courses with more focus on statistics and modeling. In mathematics departments at four-year institutions, elementary-level statistics enrollments in fall 2010 exceeded the levels for fall 2005 by about 56 percent, and enrollments have more than doubled since fall 1995 (CBMS, 2103, p. 1). The *Curriculum Guidelines for Undergraduate Programs in Statistical Science* (American Statistical Association Undergraduate Guidelines Workgroup (ASAUGW), 2014) suggested there is an opportunity for ASA to lead an effort to re-assess curricula for a variety of introductory statistics courses and that this effort might include creating model courses for students who have completed AP Statistics and those planning to major in statistics (p. 17).

SIAM's report (2014) of its second Modeling Across the Curriculum workshop questioned whether there are points of entry into the mathematical sciences besides the traditional calculus track:

- Might a freshman mathematics modeling class interest students in applied mathematics who might not otherwise choose mathematics as a major?
- Within the calculus track, is there a new approach that would improve student outcomes?

Acknowledging that points of entry and degree paths have changed, Howard Hughes Medical Institute (HHMI) President Robert Tijan said, "These days, a large number of students are arriving at college through remarkably diverse pathways. The scientific leader of tomorrow may be in a community college today or she may be a first-generation college student. Higher education should acknowledge these differences among students and create programs that offer diverse entry points and pathways to STEM degrees" (HHMI, 2015). Indeed, HHMI has recently strengthened their commitment to STEM education by offering $60 million in science education grants intended to challenge colleges and universities to increase their capacity to engage all students in science. In a significant departure from past initiatives, this competition is open to more than 1500 institutions in the United States that offer bachelor's degrees in the natural sciences, including liberal arts colleges, master's-granting universities, and research universities. Previous HHMI science education grant competitions were by invitation only and restricted to approximately 200 designated undergraduate institutions.

Statistics. The CRAFTY guide (2011) noted the importance of data analysis for many of our partner disciplines and argued strongly for an increased presence of basic statistical training in the first two years of undergraduate mathematics. According to ASAU-GW (2014), “To be prepared for statistics and data science careers, students need facility with professional statistical analysis software, the ability to access and manipulate data in various ways, and the ability to perform algorithmic problem-solving” (p. 4). The material should be motivated by a variety of examples and real data sets, including data collected by students. Entry-level courses should reflect the discipline and prepare students to take additional courses.

One of the content standards in *Beyond Crossroads* (AMATYC, 2006) recommended requiring students to collect, organize, analyze, interpret, and use data to make informed decisions. MAA (2015) recommended that students in the mathematical sciences work with professional-level technology tools (e.g., statistical packages) and acquire modest programming skills that can help them tackle ill-posed, real-world problems (p. 11). Such skills will prepare them for workplace demands since “using technology to address challenges has become a defining characteristic of work in the 21st century” (Change the Equation, 2015, p. 2). While an increased presence of statistics in the curriculum is valuable in its own right, it is also the case that exposing students to statistical modeling and simulation in their mathematics courses will enhance their computational skills (ASA, 2014, p. 7).

Modeling and computation. Modeling and computation can be used to introduce the scientific method and experimentation into mathematics courses, and both should also be seen as a means for enhancing conceptual understanding.

According to SIAM (2012), “Models are a simplification of reality, and can come in many forms. Some models are physical devices, such as a scaled-down model airplane. Other models are expressly quantitative in that they are phrased in the symbolic language of mathematics. We refer to these as mathematical models, and they can take the form of equations, algorithms, graphical relations, and sometimes even paragraphs” (p. 11). Here, we use the term “modeling” as an umbrella term referring to the creative process that can be mathematical, statistical, computational, data-based, or science-based.

The same SIAM guide recommended that departments offer modeling experiences at the entry level by developing “a first year modeling/applied mathematics course that precedes and motivates the study of calculus and other fundamental mathematics for STEM majors” (p. 4). A central reason for introducing students to modeling is to engage and retain students in STEM disciplines. Indeed, applications of fundamental mathematics, computation, statistics, and science in a wide range of STEM fields can be particularly appealing to potential majors.

Virtual experimentation is replacing many aspects of real-world implementation, and the demand for modelers is rapidly increasing. For example, in the context of airplane design or pharmaceutical testing, real-world experimentation is too dangerous, and models can be used to preserve human capital and other resources. In the context of financial markets, the complexity of the models and implied ability to run very large virtual experiments makes the use of models even more appealing. These kinds of shifts are increasing the workforce demand for graduates with modeling skills. “There are substantial opportunities for the mathematical community to attract and retain students if we can adapt to this growing opportunity” (SIAM, 2014, p. 29).

Designing curricula that integrates modeling, data science, information science, and computational science is challenging. At the second SIAM Modeling Across the Curriculum workshop, participants asserted that our community needs a common language and should eliminate jargon and academic silos. They posited that physically housing fields together, e.g., as an applied and computational mathematics department, might benefit students directly by exposing them to ways in which the fields interact. Because some introductory material is the same across fields, a single department might also realize efficiencies in staffing entry-level courses.

Some of the guides specifically called for development of stronger computational skills in mathematics and other STEM majors. The recommendations range from first year students learning simple tasks like managing electronic files and handling different types of file formats to mathematical science majors taking an introductory programming course. Some of these can be addressed in mathematics courses; one example is requiring students in mathematics courses to use software packages (e.g., MATLAB) that are standard in engineering practice.

Undergraduate statistics majors in particular should develop skills that enable them to handle increasingly complex data and use sophisticated data analysis approaches. Graduates should be facile with professional statistical software and other tools for data exploration, cleaning, and validation. They should possess the ability to program in a higher-level language (including the ability to write functions and use control flow in a variety of languages and tools such as Python, R, SAS, or Stata), to think algorithmically, to use simulation-based statistical techniques, and to conduct simulation studies. They should also be proficient with managing and manipulating data, including joining data sets from different sources in different formats and restructuring data into a form suitable for analysis. Acquisition of these skills should begin during their first two years of college.

Considerable faculty creativity may be required to fully integrate additional data-related, statistical practice, and computational skills into the curriculum, making relationships with faculty in the computational sciences and partner disciplines that teach applied statistics even more important. For faculty inexperienced in teaching these skills, professional development in this area is critical and should be considered a priority.

Connections to other disciplines. Faculty in other disciplines particularly value introductory mathematics courses that focus on skills used in their disciplines. All guides recommended employing a broad range of classic and contemporary applications that promote awareness of connections to other subjects, strengthen each student's ability to apply the course material in other contexts, and enhance student perceptions of the relevance of mathematics to the modern world.

Throughout the guides, we see the call for increased attention to the needs of other disciplines. For example, CRAFTY described chemistry students' need for early introduction of multidimensional topics. This guide criticized calculus and linear algebra courses taught with a strong theoretical focus that students view as obscure, formidable, and irrelevant to other disciplines. This is particularly problematic given that these two specific courses are foundational to many majors in the mathematical sciences and topics taught in these courses are necessary for many applications. For example, singular-value decomposition in linear algebra is a widely used technique in statistics, computer science, engineering, finance, and economics. Mathematicians are familiar with the topic, but many are unfamiliar with numerical algorithms developed the 1960s and 1970s due to the delay of their appearance in texts. Example topics appropriate for introduction in calculus range from a first look at Fourier series (the basis for signal processing algorithms) to Monte Carlo methods (a first step toward understanding Markov Chain Monte Carlo methods).

Mathematicians should seek (1) applications from partner disciplines for use in mathematics courses, (2) team teaching opportunities with faculty in partner disciplines, and (3) input from business and industry on desired workplace skills. They should collaborate with partner disciplines to explicitly translate notation and terminology among the disciplines in order to enhance students' ability to see connections across disciplines.

Communication. Students must learn to communicate complex ideas in ways that are understandable to collaborators, clients, employers and other audiences. Critical communication skills include the ability to produce accessible visualizations of material, effective technical writing, and strong presentation skills.

The relative importance of each communication skill depends somewhat on the field of study. For example, a theoretical mathematics student should be able to state problems clearly, understand mathematical readings, and communicate mathematical ideas both orally and in writing to audiences with disparate levels of mathematical sophistication. For statistics students, effective technical writing skills are of utmost importance. These types of skills develop over time and, therefore, opportunities to hone them should begin in the first two years of study.

Transitions. The first two years of college mathematics and statistics should be viewed comprehensively within the context of the transition from secondary to post-secondary education and the transition from two-year to four-year institutions. Our community must (1) articulate clear pathways between curricula driven by changes at the K–12 level and the first courses students take in college, (2) ensure students are correctly placed in entry-level courses, and (3) find ways to remove barriers facing students who attempt to transfer from one institution to another.

Appropriate placement in entry-level courses is an ongoing challenge in higher education. Despite the tremendous amount of effort and resources devoted to establishing effective placement mechanisms, many agree that our community has not solved the problem of placing students in appropriate entry-level courses. In addition, traditional placement efforts have focused primarily on accurate placement in mathematics and not in statistics because students entering college rarely had any background in that subject; this is no longer the case. In 2014, more than 184,000 students took the Advanced Placement (AP) statistics exam, a nine percent increase over the 2013 participation rate (The College Board, 2014). To put this in perspective, about 213,000 took the AP Calculus AB exam in 2014. In fact, the first administration of the AP exam in statistics drew more than any other AP exam had at that point. There has been a corresponding increase in the number of bachelor's degrees granted in statistics, a total increase of more than 140 percent since 2003 with a 21 percent increase between 2012 and 2013 (ASA, 2014, p. 4). The dramatic growth in the number of high school students completing an AP statistics course has caused some institutions to re-evaluate their introductory courses in statistics and data science. All institutions should develop courses appropriate for students who have taken AP statistics in high school, distinct from courses for those who have no previous exposure to statistics. These developments underscore the challenge of placing students in statistics courses that align with their backgrounds.

Two-year colleges are a large, growing, and increasingly important component of the United States higher education system; in fact, these institutions enroll nearly half of all undergraduates in this country (Bellafante, 2014). Indeed, President Obama recently announced an initiative to mint five million more community college graduates by 2020, a remarkable goal given that just over 77,000 associate degrees and certificates were awarded in 2011–2012 by community colleges (American Association of Community Colleges, 2015). According to *Two-Year Contributions to Four-Year Degrees* (National Student Clearinghouse Research Center, 2015), 46 percent of all students who completed a degree at a four-year institution in 2013–2014 had been enrolled at a two-year institution at some point in the previous 10 years. But many challenges surround the process of student transfer, including inadequate financial aid, the need for students to work excessive hours while attempting to carry a full load of courses, insufficient transfer advising, and the fact that too few course credits transfer from two-year to four-year institutions (The National Center for Public Policy and Higher Education, 2011). The *Curriculum Guidelines for Undergraduate Programs in Statistical Science* (ASAUGW, 2014) asserted that "[a]dditional efforts are needed to coordinate statistics instruction at the two-year college level, raise the profile of statistics majors at these institutions, and facilitate articulation agreements for transfer to four-year institutions" (p. 17).

Several states, including Arizona, California, and Texas, where many students are from traditionally underserved groups or the first in their family to attend college, will experience rapid growth in the number of high school graduates in the next decade. These states rely heavily on two-year colleges as the point of entry to post-secondary education, but their current rates of transfer are unacceptable. Failure to improve these rates, as well as their bachelor's degree completion rates, will mean that many students in these states will be unable to reach their educational goals. Consequently, the nation as a whole will face an even larger shortage of bachelor's degree holders than currently exists (The National Center for Public Policy and Higher Education, 2011).

Two-year college students who do manage to transfer to four-year institutions succeed at a slightly higher rate than students who went to a four-year institution straight out of high school, but the problem is that too few of them find the means to overcome all the obstacles standing in their way. Our community must find ways to remove these obstacles. CUPM's 2015 guide pointed to an effective program, The Illinois Articulation Initiative, which requires that changes in general education and transfer courses be coordinated jointly by the Illinois Mathematical Association of Community Colleges (IMACC) and the Illinois Section of the Mathematical Association of America (ISMAA). Additional strategies for paving the way for transitions from two- to four-year institutions are outlined in a recent Jack Kent Cooke Foundation report (Giancola & Davidson, 2015).

2.1.2 Course structure

Pedagogy. Across the guides we see a general call to move away from the use of traditional lecture as the sole instructional delivery method in undergraduate mathematics courses. The ASA (2005) asserted that, "[a]s a rule, teachers of statistics should rely much less on lecturing and much more on alternatives such as projects, lab exercises, group problem-solving, and discussion activities" (p. 9). Even within the traditional lecture setting, we should seek to more actively engage students than we have in the past.

All seven guides stressed the importance of moving toward environments that incorporate multiple pedagogical approaches throughout a program. Oft-cited examples are active learning models where students engage in activities such as reading, writing, discussion, or problem solving that promote analysis, synthesis, and evaluation of class content. Cooperative learning, problem-based learning, and the use of case studies and simulations are also approaches that actively engage students in the learning process (University of Michigan Center for Research on Learning and Teaching, n.d.). These types of pedagogies promote collaboration and provide opportunities to practice communicating ideas. A multifaceted approach to instruction is important for helping students develop flexibility in the ways they process information, and the use of diverse instructional approaches should be a strategic part of the curriculum.

A common misconception is that information is easier to process when it matches a person's preferred cognitive style (Pashler, et al, 2008). This theory, known as the "matching hypothesis" in education, suggests that "visual learners" and "auditory learners" engage best with material presented in their preferred mode. Yet, research suggests that style flexibility may actually be more important. Teaching a student to select the most appropriate style for a given situation among a variety of styles and to switch styles when necessary is a much more beneficial approach. (Kozhevnikov, 2014).

Technology. All seven guides advocated using technology to enhance student learning. Pedagogical innovations are often driven by advancing technologies. Faculty and students should learn to use technology and become intelligent consumers of the answers it provides. Technology can be used as a tool for developing conceptual understanding, analyzing data, and for solving problems. It can strengthen students' problem-solving skills by encouraging them to employ multiple strategies (graphical, numerical, algebraic) in the process. Students can use technology to perform extensive computations (e.g., with large data sets in statistics or on large systems in linear algebra) that are unrealistic by hand and to generate visualizations such as graphs, histograms, and other diagrams that enhance comprehension.

> [T]echnology can promote students' exploration of and experimentation with mathematical ideas. For example, students can be encouraged to ask "what if?" questions, to posit conjectures, to verify or refute them, and to use technology to investigate, revise, and refine their predictions. Specific examples include studying the effects of manipulating parameters on classes of functions and fitting functional models to data (MAA, 2004, p. 24).

Using specialist software (e.g., R or MATLAB) for teaching can also provide students direct experience with tools routinely used in the workplace. Indeed, *Beyond Crossroads*

(AMAYTC, 2006) recommended using technology throughout the curricula to help students discover properties, develop concepts, consider multiple perspectives, and to give students experience with the technology skills they will use routinely in the workplace.

2.1.3 Workforce preparation

Mathematical sciences departments play a major role preparing a mathematically- and scientifically-literate workforce. While we acknowledge there is no one-size-fits-all solution, we also recognize the need for structures to catalyze widespread adoption of curricula and evidence-based pedagogies that are (1) geared toward developing a broad base of intellectual skills and competencies to better prepare students for the workforce and (2) endorsed by the mathematical sciences community.

In response to rapidly changing workforce needs, departments should establish advisory committees that include representatives from business, industry, and government to regularly engage in conversations about the expectations of prospective employers. Departments must engage partners from inside and outside academia in STEM and non-STEM fields to ensure that the applications taught are realistic and that the skills students take from courses are valued by stakeholders. Our community must prepare graduates who are career-ready and focus intentionally on workplace skills early in their programs. Even in the first two years of college, students must have opportunities to improve their speaking and writing skills, to work with data, and to engage in open-ended inquiry. Data skills, in particular, are increasingly attractive to employers. According to *Fueling Innovation and Discovery* (NRC, 2012):

> The mathematical sciences contribute to modern life whenever data must be analyzed or when computational modeling and simulation is used to enable design and analysis of systems or exploration of "what-if" scenarios. The emergence of truly massive data sets across most fields of science and engineering, and in business, government, and national security, increases the need for new tools from the mathematical sciences (p. 2).

STEM and non-STEM graduates with marketable skills contribute to the "common good" by advancing national priorities that are in the best interests of all citizens. Mathematical competencies lead to higher-paying careers and, thus, can play a profound role in students' economic mobility (Haskins, Holzer, & Lerman, 2009).

In its conclusion, *Beyond Crossroads* (AMAYTC, 2006) highlighted the critical importance of partnerships with other disciplines as well as with business and industry employers. It asserts:

> Technical mathematics courses and programs should be developed in collaboration with faculty from other disciplines and business and industry representatives to identify and address the mathematics content needs of specific program employers. Content in two-year technical mathematics courses should be selected because of its application to a specific technical field and the needs of specific employers. It should also be at a level equivalent to mathematics courses that transfer to four-year institutions (p. 44).

Partnerships inside the institution. Faculty in disciplines outside mathematics rarely ask their students to find the equation of the line that passes through two given points. But social scientists, for example, will expect students to recognize a linear pattern in a set of data, interpret the parameters of the line of best fit, and use the equation of the line to answer questions in the context of a real-world scenario. Mathematical sciences departments should be aware of applications used in other disciplines and adjust their general education and introductory courses accordingly (MAA, 2004).

In discussing the role of modeling in the curriculum, SIAM (2102) distinguished between mathematical models and mathematical modeling. The former refers to the presentation of finalized models that primarily expose students to applications of abstract mathematics (e.g., exponential functions to mimic population growth) without attention to derivation or assumptions behind the models. The latter attends to "the creative process by which the model is developed" (p. 11), including construction and evaluation criteria. Both instructional approaches are strengthened when mathematics faculty members draw on partnerships with faculty in other disciplines. SIAM asserted:

> By working across campus, faculty can develop strong ties. If done well, modeling will help bind applied mathematics to the rest of STEM. We recommend constant outreach, continually making new connections. An additional advantage of community building is that it allows for better leveraging of existing infrastructure and resources. Intellectual diversification will also make mathematics a central theme and not allow the modeling to become dominated by a specific scientific discipline (p. 13).

Partnerships outside the institution. Departments should periodically consult with representatives from local businesses and industries and use this input to ensure course content remains relevant.

Each department should be well-informed about the paths their graduates choose and should use that information to inform decisions about their courses and programs. AMATYC called on two-year institutions to implement methods for tracking their students after they graduate or transfer and consider whether or not their mathematics courses and programs appropriately address students' educational and career needs. Echoing this, ASA (2014) suggested departments survey both graduates and employers to gather information on the career paths of the growing number of statisticians in the workforce and use this information to guide curricular decisions.

An important aspect of workforce training is ensuring students, parents, guidance counselors, teachers, and administrators are aware of the vast array of career options available to mathematics and statistics graduates. Students should be well-informed of the possibilities before they choose a college major. Faculty members teaching courses in the first two years of college should also understand which skills are needed for various career paths.

> A math degree can prepare students for high-paying and influential careers in finance, insurance, risk management, operations research, computational science and engineering, signal, image, and natural language processing, bioinformatics and computational biology, information science, and machine learning, to name a few (SIAM, 2012, p. 13).

Other career fields that could be added to this list are advanced manufacturing, national security, STEM education, and data science as used, for example, in climate science, international development, and health care.

2.1.4 Faculty development and support

Higher education as an industry invests little in the development of its front-line instructional staff, but ongoing training and support are necessary in all mathematical fields, perhaps especially in emerging areas in the mathematical sciences. Given the limited number of programs run by professional associations that have been instrumental in developing mathematics faculty (e.g., MAA's Project NExT and AMATYC's Project ACCCESS), it is no surprise that the need for developing both full-time and contingent faculty is highlighted in all the guides. In order to adapt curricula, develop new courses, incorporate pedagogical tools, and use technology effectively for instruction, faculty members require training, resources, and rewards for their efforts.

ASA's *Curriculum Guidelines for Undergraduate Programs in Statistical Science* (ASAUGW, 2014) articulated the need this way:

> A considerable barrier to implementing these guidelines is the lack of materials related to data science topics. Efforts to pull together activities, projects, sample syllabi, and model courses as well as training are needed to ensure that faculty have the appropriate skills to teach aspects of this new curriculum (p. 11).

As AMS pointed out in its response (www.ams.org/policy/govnews/pcast-statement) to the PCAST report (2012), it is essential that mathematicians become engaged in planning and teaching courses that form the foundation of STEM education. Notably, this will require training for faculty in priority areas emphasized by ASA that have attracted attention from stakeholders outside the academy, including from the federal government with their recent focus on big data (Kalil, 2012). Faculty training in these areas merits substantial investment as does training for all faculty in the mathematical sciences. In a recent white paper, ASA (2014) pointed to

> … scientific challenges facing many broad areas being transformed by Big Data — including healthcare, social sciences, civic infrastructure, and the physical sciences — and describes how statistical advances made in collaboration with other scientists can address these challenges. We recommend more ambitious efforts to incentivize researchers of various disciplines to work together on national research priorities in order to achieve better science more quickly (p. 1)

AMATYC (2006) highlighted the need to "provide support for faculty in seeking outside funding to support the technology appropriate for the curriculum" (p. 45). And SIAM (2012) urged administrators and other funders to "provide seed grants for faculty to develop, implement and evaluate new approaches to the high school-college math transition for STEM majors" (p. 4).

It is critical that faculty engage with efforts to improve teaching and strengthen curricula, including curricula in the first two years for both majors and non-majors. Faculty members and departments are less likely to respond to this call if they are not rewarded for their efforts, and they will be unable to respond in a meaningful way if institutions fail to provide ongoing professional development and support. Administrators who understand the critical role that the mathematical sciences play in economic mobility are more likely to invest in faculty development, and it is up to mathematicians and statisticians to communicate this vital role.

Departments and institutions must also engage in an ongoing cycle of self-reflection and revision. The MAA (2015) called on departments to continually strengthen courses and programs to better align with student needs and to assess the effectiveness of such efforts. Curricula and programs require updates if they are to maintain relevance to current job opportunities for professional mathematicians, statisticians, and other graduates who possess strong STEM skills. Mathematical sciences departments and institutional administrators should support and reward faculty efforts to improve teaching and to strengthen curricula.

SIAM (2012) emphasized that modernizing undergraduate curricula cannot be delegated to a few pioneers:

> Sustainability is a big challenge. Many past efforts and programs have not survived on account of the founders leaving, retiring, or burning out. Therefore, we recommend getting many people involved so that efforts are integrated into the culture and become a community effort, not something being driven by one or two people (p. 13).

They went on to say that the greater the number of faculty members involved and the greater the investment in infrastructure, the more likely the program is to survive in the long run. MAA (2004) also addressed the issue of sustainability:

> No program will long survive if it represents the work of a single individual. For long-term sustainability, initiatives must be team efforts, with faculty in supporting roles who can be prepared to expand or take over the leadership of the program. At institutions that require research for promotion and tenure, untenured faculty should not be expected to take on roles that would seriously hamper their scholarly development. However, means should be found to involve all faculty in improving the curriculum and its instruction. This is especially true for younger faculty who often bring enthusiasm and openness to innovative ideas (p. 25).

The mathematical sciences community must foster an institutional culture that adequately encourages and values work on the problems highlighted in this report without overstretching the goodwill and enthusiasm of faculty members. Large-scale changes are needed and the Coalition for Reform of Undergraduate STEM Education (Fry, 2014) recommended convening "national summits focused on the development of new systems and approaches for faculty incentives, including promotion and tenure, and faculty development programs that are organized to achieve optimal impact on strengthening teaching and learning" (p. 7). The MAA (2015) asserted:

> Failing to align rewards with department needs for renewal and reform leads to stagnation. A department that values faculty involvement in undergraduate research, interdisciplinary courses, experimental coursework, and new pedagogy should assure

that suitable credit is awarded in annual reviews. Deans, department chairs, and colleagues should recognize that colleagues who risk doing innovative work deserve both encouragement and support in the planning and execution stages of projects and appropriate rewards when they come up for periodic review (p. 57).

2.2 Other important themes

In addition to the four themes identified above as common to all guides, there are a number of other areas that warrant considerable attention from the mathematical sciences community, but that do not appear in all the guides. We view increased attention to these as critical for improving undergraduate mathematical sciences education and want to highlight them as additional areas of focus. Indeed, they are compelling and, together with the common themes in Section 2.1, form the basis of our recommendations for moving forward that are outlined in Chapter 3. As with the common themes, the topics here are interdependent.

Student diversity. The fact that our community has been unable to attract and retain a diverse student population in the mathematical sciences is a dreadful shortcoming that must be remedied.

The underrepresentation of minorities in advanced mathematics courses is an ongoing problem. Walker (2007) noted that, while there has been some improvement since Stiff and Harvey (1988) called the mathematics classroom one of the most segregated places in the United States, upper-level mathematics classes remain predominantly white. Black and Latino students are less likely than Asian American and white students to complete advanced high school mathematics courses (National Center for Education Statistics, 2014) and the performance gap in mathematics is evident as early as fourth grade (Fry, 2014, p. 3).

The abysmal retention rates for both women and minorities is exacerbated by structural features of undergraduate programs in the mathematical sciences:

> Minority students and women who start below the standard entrance point (calculus) for majors may be particularly reluctant to continue in mathematics. … Even when a curriculum is structured to allow alternative entry points to the major, many students need advisors, coaches, professors, and peers to help clear the way (MAA, 2015, p. 65).

The MAA's national study of Calculus I, *Characteristics of Successful Programs in College Calculus* (MAA, n.d.), highlighted some important differences between men and women who study calculus in college. For example, it found that women were almost twice as likely as men to choose not to continue in calculus, even when Calculus II was a requirement for their intended major (Bressoud, 2011).

Singling out minority students and women might be misinterpreted as a presupposition that they are less capable or less prepared than other groups, but that is not our view. Rather, we contend that faculty and administrators should implement systemic changes that expand opportunities and remove barriers to success for all students.

Faculty and administrators at four-year institutions should also remain cognizant of the increasing prominence of two-year colleges as gateways to four-year degrees since these institutions play a significant role educating minority students. For example, 59 percent of all Native American and 56 percent of Hispanic students in higher education attend a community college (American Association of Community Colleges, 2014). The Department of Education's (2007) *Persistence and Attainment of 2003–04 Beginning Postsecondary Students: After Three Years* told a similar story.

Student mobility. A nontrivial proportion of our students attend more than one institution, a trend somewhat more pronounced among two-year college students. The mobility rate for public two-year institutions was 11.5 percent in 2012–2013, compared to 8.7 percent at public four-year institutions (American Association of Community Colleges, 2014). That said, students are transferring among all types of institutions. Roughly 3.6 million students entered college for the first time in the fall of 2008 and, over the past six years, have transferred 2.4 million times, "ricocheting between two- and four-year public and private colleges, often across state lines" (The Chronicle of Higher Education, 2015). An increasingly common path is for students to start at a four-year institution and transfer to a two-year institution. In fact, only about half of the students who transfer out of a four-year institution transfer to another four-year institution; the other half transfer to a two-year institution.

The high mobility rate exacerbates challenges for students from lower economic sectors. According to The National Center for Public Policy and Higher Education (2011), 44 percent of low-income students enroll in two-year college after high school, compared to 15 percent of high-income students. Because transferring is more common for two-year college students, mobility appears to be most prevalent among low-income students.

These facts highlight the need to work across sectors within higher education, as well as across states, to improve student success. Students are consuming courses differently than they did in the past, and the increased mobility across institutions necessitates a framework of shared responsibility for students across all of higher education. Extending our discussion on transitions, articulation and transfer agreements help ensure continuity of programs of study, fewer repeated courses, and timely completion, saving both time and money. For example, the University of California system has agreements with all 112 California two-year colleges specifying the courses that will receive bachelor's degree credit and those that may be used to fulfill general education requirements (University of California, 2014). Even within the context of a well-established system like this, California students face hurdles as they transfer from a two-year to a four-year institution. Some of their challenges are related to mathematics courses; for example, the vast majority of two-year college students are placed into a developmental course and some may take up to four such courses. Because failure rates are so high in developmental courses, a sequence of these courses exponentially reduces the number of students moving into credit-bearing courses; in fact, most students deemed unready for a credit-bearing mathematics course will never graduate. There are additional challenges unrelated to mathematics courses; for example, four-year institutions in California have a limited number of admissions slots and therefore have little capacity to enroll more transfer students without decreasing their freshman enrollment (Burdman, 2015). Agreements within states, like the California example, as well as multi-state agreements are in jeopardy due to state budget cuts.

Contingent faculty. The trend away from the tenure-track model is linked to the changing economics of the higher education enterprise. A large and growing number of contingent faculty — any faculty member in a non-tenure-track position, whether part-time or full-time, short-term or long-term — is currently teaching our courses. Broad reform will require professional development opportunities for all faculty members, including contingent faculty who often have limited access to such activities. Deans and other administrators need to be a part of this conversation.

Non-tenure-track positions of all types now account for 76 percent of instructional staff appointments in American higher education (AAUP, n.d.). Of special concern is the large number of such faculty at two-year colleges, institutions which enroll nearly half of all undergraduates in this country. At some two-year colleges, as many as two-thirds of the faculty members are in part-time positions (Bellafante, 2014). Overall, 46 percent of all two-year college mathematics classes were taught by part-time faculty in 2010 (CBMS, 2013, Table S.4, p. 13). Two-year colleges in particular face the challenge of high turnover rates among part-time faculty, and such turbulence demands action to support contingent faculty in ways that will benefit these individuals and their students. The various challenges part-time faculty members face and strategies to address their professional development are discussed in detail in *Beyond Crossroads* (AMATYC, 2006).

Contingent faculty members at all types of institutions are important contributors to the teaching mission yet these colleagues often do not enjoy the monetary or other supports that tenure-track faculty do. Nevertheless, faculty working conditions and the educational experiences of our students are inextricably linked. Full-time, non-tenure-track faculty may not become fully engaged in the life of the department with regard to non-instructional contributions (e.g., advising and mentoring students, curriculum design). Developing contingent faculty so that curriculum initiatives can be successfully implemented benefits these colleagues as professionals as well as the students in their classrooms.

Non-Tenure-Track Faculty in our Department (The Delphi Project on The Changing Faculty and Student Success, 2012) articulated the factors that have led to a majority of faculty being hired off the tenure track, the impact on teaching and learning, and suggestions to improve policies and practices that affect non-tenure-track faculty and create better conditions for student learning.

Future teachers. Attending to the preparation of future K–12 mathematics and statistics teachers is critical to the future of our profession, and mathematics departments should work with schools of education to ensure pre-service teachers are well-prepared for the classroom. The specialized knowledge required for teaching mathematics is distinct from the mathematical knowledge needed for other mathematically-intensive occupations and professions (Thames & Ball, 2010). The CUPM guide (MAA, 2015) asserted “that a traditional liberal arts major in mathematics is neither necessary nor sufficient preparation for teaching high school mathematics (unless it is followed by graduate training in mathematics education).” *Towards Excellence: Leading a Doctoral Mathematics Department in the 21st Century* (AMS, 1999) went as far as to say that “if K–12 mathematics education in the U.S. deserves criticism, … then a share of the blame falls to those university mathematicians who should be playing an important role in the preparation of teachers but are not" (p. 24). Additionally, the same report predicted that colleges and universities will be called upon to play a larger role in the important business of improving mathematics education. This requires more mathematicians to take a role in the continuing education of teachers and to make a contribution to the public discussion of what is taught and how it is taught. Indeed, engagement is imperative, since "[f]aculty members of mathematics and statistics departments at two- and four-year collegiate institutions teach the mathematics and statistics courses taken by prospective and practicing teachers. Their departmental colleagues set policies regarding mathematics teacher preparation." (CBMS, 2012, p. xiii).

Fortunately, there is evidence that we are making progress alleviating the historical tension between schools of education and mathematics departments. The mathematical sciences community has re-embraced its role in the preparation and professional development of K–12 teachers, as noted in *The Mathematical Education of Teachers II* (CBMS, 2012):

> In 1893, the Committee of Ten, composed of presidents of Harvard and other leading universities, led the creation of influential school curriculum guidelines. Among the writers were Simon Newcomb and Henry Fine, both future presidents of the American Mathematical Society.… However, for a variety of reasons, both internal and external to the U.S. mathematics community, concern for school mathematics and its teachers did not retain similar prominence among mathematicians during much of the twentieth century.… Over the past decade, this situation has begun to change (p. 7).

For example, Maryland has an informal network of mathematics specialists in the public school system and faculty from mathematics departments in higher education institutions that meets at least once a semester to discuss issues of common concern (MAA, 2015, p. 57).

Graduate students. Graduate students are important contributors to the teaching mission of many universities, in much the same way that contingent faculty are. Those who pursue academic careers will be the faculty of the future, but, historically, the mathematical sciences community has provided very little training focused on preparing them to teach. While research universities play a unique and significant role in the preparation of these future faculty members, they in turn hire only a small proportion. According to *Towards Excellence* (AMS, 1999), less than 20 percent of new Ph.D.s find employment at Ph.D.-granting institutions, so the remainder of those who pursue careers in higher education work in other types of institutions and spend much of their time teaching undergraduates. Vélez et al (2014) reported that 32 percent of 2012–2013 Ph.D. recipients found employment upon graduation in Ph.D.-granting departments but almost three-quarters of them were in post-doctoral — and hence not longterm — appointments. These data indicate that our community has made little progress over the past 15 years. Attending to graduate student preparation for teaching has far reaching benefits beyond the institutions at which they serve as teaching assistants, to students at institutions where they will spend their careers.

AMS (1999) pointed to some promising examples such as the Preparing Future Faculty initiative sponsored by the Pew Charitable Trust. This program works with graduate students across disciplines to help them develop expertise in teaching and expose them to the professional environment at a variety of institutions, including two-year colleges, liberal arts institutions, and comprehensive universities. MAA has a project underway to create infrastructure to support high quality, teaching-related professional development for graduate students. A primary goal of this project is to support faculty who wish to begin such a program but who may find it challenging to locate instructional materials or other faculty with experience on which to draw.

Failure rates. The high rate of failure in post-secondary mathematics classes is an embarrassment to our profession. It is a major contributor to increased attrition rates, and it lengthens time to degree at all types of post-secondary institutions. Mathematics courses are the most significant barrier to degree completion in both STEM and non-STEM fields. For example, each year only 50 percent of students attain a grade of A, B, or C in college algebra, and fewer than 10 percent of those who pass go on to enroll in a calculus course (Ganter & Haver, 2011, p. 49). These data, together with the issues surrounding college algebra discussed earlier, indicate that the role of college algebra should be reconsidered. Developmental mathematics courses also serve as barriers to degree completion. According to a recent New York Times article, for students not prepared for college-level mathematics, passing a developmental class,

> which teaches math that most affluent children study in eighth or ninth grade, is required for graduation and the ascent to four-year programs. But at community colleges across the country, the basic math requirement has been a notorious hindrance to advancement. More than 60 percent of all students entering community colleges must take what are called developmental math courses, according to the Carnegie Foundation for the Advancement of Teaching, but more than 70 percent of those students never complete the classes, leaving them unable to obtain their degrees (Bellafante, 2014).

High failure rates more generally point to the need for the mathematical sciences community to remove barriers to success and enhance student perseverance, confidence, and ability to use mathematics to solve problems. In addition, supports are needed to help students make connections between the content of their courses, career choices, and pathways through curricula, and to overcome math anxiety and negative feelings toward quantitative fields.

Developmental courses. According to the most recent data from CBMS, approximately 57 percent of students at two-year colleges and 23 percent at four-year institutions take at least one developmental mathematics course (CBMS, 2010). In the face of pressure to accelerate the remediation process, new models have been developed, including options for self-remediation or that incorporate "just in time" remediation within the context of a credit-bearing course. For example, the University of Illinois uses ALEKS® to provide students with a tool for self-remediation as preparation for placement into calculus (www.math.illinois.edu/ALEKS/).

Calculus. Calculus is central to many disciplines and completing at least one calculus course is a requisite for most STEM degrees. But a convergence of factors — increasing numbers of inadequately prepared students taking calculus, declining resources for faculty teaching calculus, changing demands of partner disciplines, and the rush to calculus in high school — has led Bressoud (2015) to declare that calculus is "in crisis." The joint statement of MAA and NCTM (2012) on calculus asked faculty to redesign college calculus curricula in response to the ubiquity of calculus in secondary schools and encouraged faculty in higher education institutions and high school teachers to work together to re-envision the role of calculus. Students should only take calculus when they are adequately prepared, and the calculus they take should be attentive to the needs of partner disciplines.

Technology-enabled delivery models. The prevalence of technology provides increasing opportunities to enhance existing courses and to offer courses in alternative formats. Flipped and blended classes, classroom management systems (e.g., Blackboard, Moodle), web-based interactive systems (e.g., WeBWorK), adaptive learning platforms, massive open online courses (MOOCs), and other distance learning platforms are examples of ways in which faculty are using technology to improve student learning.

MOOCs and other online courses in the mathematical sciences provide more options to students for required courses and additional opportunities to enhance their education with more advanced courses not offered by their home institution. Flipped and blended classrooms can be used to engage students more actively in the classroom. Technology can be used to enable innovation to strengthen teaching and learning in the classroom, and also to help control costs in education; e.g., by utilizing classroom management systems, or by offering MOOCs.

Yet, multiple challenges exist in this arena. For example, there is a dearth of empirical evidence on the effectiveness of such delivery models. Challenges related to online and blended classes include ensuring standards are held at the same level as those for face-to-face classes and maintaining integrity of the student assessment process. Specific to MOOCs is that they seem to work best for people who have already "learned how to learn" (Zorn, et al, 2014, p. 40).

Assessment. For the purposes of this report, we define "assessment" as the process of gathering and using empirical data to refine programs and improve student learning. Assessment can be formative or summative, and while we focus on assessment at the the program level, it can be focused on individuals, courses, departments, institutions, or larger systems. While assessment is an important ingredient in improving education, our community must be vigilant to ensure that assessment systems do not significantly inhibit faculty willingness to change and take risks. When new programs are adopted or adapted, data should be collected and used for ongoing evaluation and improvement purposes. Departments must use compelling measures of program effectiveness and should be responsive to the data they collect.

Scaling. Related to the issue of sustainability discussed previously is the broader issue of scaling. There are many examples of curricular and pedagogical work on individual campuses that merit careful study and emulation, but successfully scaling-up such initiatives will require our community to address the following questions:

- How do we help open channels of communication within the mathematical sciences community to raise visibility of these efforts?

- How do we involve, within a department or program, a broader range of faculty and students in a given project?
- How do we transfer initiatives to other institutions?

In a presentation given at a 2013 *College Changes Everything* conference, Dede (n.d.) describes scaling-up as "adapting an innovation successful in some local setting to effective usage in a wide range of contexts." We note that his use of the word "adapting," as opposed to "adopting," fits with our belief that there is no one-size-fits-all approach. Any adoption of curricula or pedagogy will require tailoring to local needs and ongoing monitoring of local data to ensure its effectiveness remains intact, thus, adapting to local needs. Most important is attending to integrity above fidelity of implementation.

3 Moving Forward

The May 2015 *Common Vision* workshop provided an opportunity for participants to collectively reflect on an initial draft of this report, brainstorm together, and propose future action. The group outlined explicit ways in which faculty members, deans, and other administrators can create an environment that supports improvements in undergraduate mathematical sciences education. They range from short-term, low-cost initiatives to long-term, high-cost initiatives. We summarize these proposals below, and we hope these summaries will (1) guide federal agencies in targeting future funding for undergraduate education in mathematics and statistics and (2) prompt members of our community to launch initiatives to improve teaching and learning in the mathematical sciences.

3.1 Short courses and workshops

Our community recognizes the value of meeting in person to work on professional development, community building, and other activities; e.g., short courses and workshops are common in the mathematical sciences. *Common Vision* participants identified several areas of focus for future gatherings and suggested workshops and short courses be held in conjunction with major professional society meetings (e.g., the Joint Mathematics Meetings) or online. Specific workshops proposed include:

- A workshop to bring together department leaders of comparable departments to share practices for improving undergraduate student outcomes. These leaders are under increasing pressure to improve student success and persistence rates, and they would benefit from networking with peers about issues surrounding evidence-based initiatives (e.g., what is working, implementation strategies, and solutions to associated challenges).
- A series of workshops on active learning techniques for precalculus and calculus aimed at changing the way the first two years of college mathematics is taught.
- A workshop devoted to providing support for departments implementing curricular changes. For example, a workshop for department leaders and other faculty members from small mathematics departments implementing recommendations in the ASA and MAA guidelines for teaching statistics.
- A workshop focused on developing guidelines endorsed by one or more professional associations on evidence-based practices for training graduate students to teach. Our graduate students are the faculty of the future, yet none of the professional associations

has published a guide for training graduate students to teach. Initiatives such as the aforementioned Pew and MAA projects should be used to inform such a guide.

- A workshop bringing together college faculty and high school teachers to improve student training in modeling, as outlined in SIAM's Modeling Across the Curriculum guides.
- A workshop to build on the successes of the first two Modeling Across the Curriculum workshops. Such a workshop might focus on effective ways to educate students at the intersection of mathematical modeling, data science, information science, and computational science. A product of this workshop might be a series of lesson plans for teaching topics in this intersection. Workshop participants would be employed in these fields and be familiar with applications from a broad range of areas (e.g., climate modeling, business analytics, machine learning, bioinformatics).

3.2 Course and curriculum development

Common Vision workshop participants identified the need for additional focus on college algebra and other precalculus courses, on first year STEM courses, and on creating a consensus document on "pathways." They noted that most paths into and through STEM majors are front-loaded with calculus. Indeed, the NSF-funded INGenIOuS report (Zorn, et al, 2014) identified the development of multiple curricular pathways as an important next step for the mathematical sciences community. *Common Vision* participants specifically identified the need for one such pathway that integrates computing and statistics. Examples of a variety of pathways could serve as the foundation for a consensus document.

Another proposal aimed at enriching curricula focused on a faculty externship program, during which faculty members spend time embedded in business, industry, or government sectors. These might take place during the summer or a sabbatical, or might be during a regular semester/trimester/quarter. This kind of experience would enhance faculty knowledge and understanding of jobs available to graduates, and the input they would gather from colleagues in these non-academic sectors would serve to inform curricula development and revision. Departments and institutions should support faculty in such programs; a similar recommendation is also found in the INGenIOuS report (Zorn, et al, 2014). Macalester College is one school that has supported sabbaticals for faculty embedded in business and government, and these experiences have served to inform mathematics curricula revisions.

3.3 Policy initiatives and public relations

One of the goals of *Common Vision* was to bring together members of different professional associations from different types of post-secondary institutions to consider current undergraduate curricula. Progress toward modernized curricula will require not only a united, collective voice, but also advocates who are respected in the various sectors within our community as well as sectors external to our community. *Common Vision* participants outlined a major web-based project to produce videos aimed at educating the public about basic mathematics and statistics concepts. One external challenge our community faces is that there are many policy makers and other key stakeholders who talk about contemporary topics like “big data” and make related policy decisions without any real understanding of mathematics, statistics, or data science.

In a number of states (e.g., Ohio, Georgia), statewide commissions have been established to bring together chairs of mathematics departments from across the state system to create a statewide agenda for improving student success in undergraduate mathematics. Some states have made substantial progress on a shared understanding among leaders in higher education of curricular issues such as course content and transfer credit. Uri Treisman, distinguished fellow at the Education Commission of the States and *Common Vision* leader, has been a major force in leading these efforts, but they have grown beyond what one person or one organization can do. The United States Department of Education has data about institutions which have developed curricula and transfer assurance guides detailing competencies that must be included in all syllabi. Web tools already exist to facilitate transfer; e.g., collegetransfer.net provides access to transfer agreements between specific institutions. *Common Vision* participants emphasized the need to build trust among institutions for the benefit of transfer students. The breadth of institution-type represented by *Common Vision* participants is a strength of this project that might serve as a way to prompt the formation of additional statewide commissions to expand on the work done in Ohio and Georgia. These in turn might provide a national platform for policy advocacy on educational issues.

Common Vision participants also proposed establishing a Joint Mathematics Meeting (JMM) invited lecture on the undergraduate experience. Two AMS-MAA Invited Addresses take place annually at each JMM, and workshop participants proposed that one of these two highlight an education or teaching-related topic. Such a move would represent a strong visible commitment by both the AMS and MAA to this aspect of our profession.

Another proposal suggested establishing a higher education policy fellows program at the state level similar to the American Association for the Advancement of Science (AAAS) Congressional Fellows Program at the federal level. Such a state-level program might be run through CBMS, for example. State governments play a central role in higher education, and we should have more faculty members engaged in the policy process. A state-level program would be considerably less expensive than the AAAS program at the federal level; e.g., faculty would not be required to temporarily relocate in order to participate.

3.4 Center for the Advancement of Mathematical Sciences Education

Common Vision participants were intrigued by the idea of a Center for the Advancement of Mathematical Sciences Education to support institutional change toward improving teaching and learning in core undergraduate mathematical sciences courses. There was lively debate about whether a physical or virtual center would be most viable and recognition that such a center could facilitate many of the recommendations made during this workshop.

An important function of such a center might be to serve as a clearing house for resources. It could provide easily accessible information for faculty and administrators about evidence-based innovations in the mathematical sciences involving curricula, classroom practices, institutional changes, two- and four-year institution collaborations, state policies and initiatives, national initiatives, and professional association efforts. In 2012, SIAM urged "the NSF to fund and/or house a central repository that provides the STEM community a well-maintained information resource of past and existing programs" (SIAM, 2012, p. 14). Indeed, several professional associations have developed online repositories in the past, but experienced varying degrees of success in engaging the community with these tools. The associated challenges include quality control, the ongoing need for updates, and the time required for maintenance. The proposed center would help address these challenges.

This center might also host themed semesters similar to these of the NSF-funded mathematical sciences institutes (e.g., on preparing students to work with big data, on modeling applications in the sciences) and run a consultants program for departments (e.g., to help departments accelerate adoption of instructional practices that actively engage students in the classroom, institute a data science curricular pathway, or increase minority representation in the major). *Common Vision* participants recommended cross-sector engagement with the center (community colleges, liberal arts, master's granting, research-intensive) and asserted that a center could effectively create and support mechanisms and frameworks for cross-institutional collaboration. A key to success will be having prominent mathematical scientists (e.g., Fields medalists, National Academy of Science Fellows) endorse the center.

The full vision for such a center is that it pursue its mission through a variety of strategies:

- Develop and lead assessment efforts to identify essential evidence-based elements of successful programs.
- Create and maintain a repository of programs and initiatives that have demonstrated potential for increasing student success.
- Assist departments and programs in outlining their objectives for change.
- Offer programs designed to strengthen curricular offerings, help faculty expand and hone pedagogical approaches, and develop additional leaders.
- Foster activities (special sessions, panel discussions, plenary talks, etc.) at association conferences that focus on the mathematical training of STEM majors and disseminate resulting products (e.g., videos) to the broader mathematical sciences community.
- Communicate with other STEM disciplinary associations and higher education associations to improve alignment of the center's goals with those of partner disciplines.
- Implement communications strategies to engage the mathematical sciences community at large in the quest to improve undergraduate instruction, to inform external constituencies (e.g., funding agencies) of current progress and the overall vitality of the mathematical sciences, and to attract partners who can provide new ideas and resources.

While intrigued by the idea, *Common Vision* participants did not reach a consensus on the value of such a center. Some participants who favored the idea also acknowledged that there are already many resources available from professional associations and that the community might be better served by more closely coordinating existing efforts of the associations. A center might drain resources from established projects that are worthy of support and improvement. However, the majority of *Common Vision* participants agreed the community would benefit from more collective efforts such as establishing the proposed center.

4 Conclusion

The goal of the *Common Vision* project was to effect changes in undergraduate mathematical sciences education in order to expand scientific knowledge and maintain a viable workforce in the United States. These changes will necessitate efforts to (1) update curricula, (2) scale up the use of evidence-based pedagogical methods, and (3) establish stronger connections with other disciplines.

In many ways, *Common Vision* built on the work of the INGenIOuS project, and we want to leave you with a call to action from their report (Zorn, et al, 2014):

> We acknowledge that changing established practices can be difficult and painful. Changing cultures of departments, institutions, and organizations can be even harder. But there is reason for optimism. In mathematical sciences research we are always willing, even eager, to replace mediocre or "somewhat successful" strategies with betters ones. In that open-minded spirit we invite the mathematical sciences community to view this call to action as a promising opportunity to live up to our professional responsibilities by improving workforce preparation (p. 25).

Common Vision succeeded in bringing together leaders from AMATYC, AMS, ASA, MAA, and SIAM to work toward the goal of improving undergraduate education. By reaching out to members of these five associations, we hoped to galvanize the mathematical sciences community around a modern vision for undergraduate programs and to spur grassroots efforts within the community. Indeed, *Common Vision* participants worked together to identify common themes in existing curricular guides that have been endorsed by these associations and to propose a path forward for continued collaboration.

A primary point emphasized by all the guides is that ***the status quo is unacceptable***. Change is unquestionably coming to lower-division undergraduate mathematics, and it is incumbent on the mathematical sciences community to ensure it is at the center of these changes, not on the periphery. We hope other individuals and groups will come alongside us in this effort, capitalize on the momentum we have built and goodwill we have established, and move this effort forward into a second phase focused on implementation initiatives.

Glossary

Assessment. Assessment is the process of gathering and using empirical data to refine programs and improve student learning. Assessment can be formative or summative, and while we focus on assessment at the the program level, it can be focused on individuals, courses, departments, institutions, or larger systems.

Contingent faculty. A contingent faculty member is any faculty member in a non-tenure-track position, whether part-time or full-time, short-term or long-term. The AAUP definition includes those employees with the title of adjunct, postdoc, TA, clinical faculty, lecturer, and instructor.

Developmental. Developmental courses are non-credit-bearing courses taken at an institution of higher education.

Flipped classroom. A flipped classroom is a model in which students gain first-exposure learning prior to class and focus on the processing part of learning (e.g., synthesizing, analyzing, problem-solving) in class.

Mathematical sciences. The "mathematical sciences" is an umbrella term for a collection of fields that certainly includes applied mathematics, theoretical mathematics, computational mathematics, operations research, statistics, and mathematics education research.

> The "mathematical sciences" must be defined very inclusively: The discipline encompasses a broad range of diverse activities whether or not the people carrying out the activity identify themselves specifically as mathematical scientists.
>
> This collection of people in these interfacial areas is large. It includes statisticians who work in the geosciences, social sciences, bioinformatics, and other areas that, for historical reasons, became specialized offshoots of statistics. It includes some fraction of researchers in scientific computing and in computational science and engineering. It also includes number theorists who contribute to cryptography and real analysts and statisticians who contribute to machine learning. And it includes as well operations researchers, some computer scientists, and some physicists, chemists, ecologists, biologists, and economists who rely on sophisticated mathematical science approaches. Many of the engineers who advance mathematical models and computational simulation are also included (NRC, 2013).

Mathematical models and modeling. Mathematical models are simplifications of reality that most often take the form of equations, algorithms, and graphical relations. We use the term “modeling” as an umbrella term to refer to the process that results in a mathematical model, and which can be mathematical, statistical, computational, data-based or science-based.

Pathways. This term “pathways” is intended to encompass pathways into majors in the mathematical sciences, pathways through these majors, and pathways through the general education requirements.

Two-year colleges. We intend the term “two-year colleges” to comprise all two-year post-secondary institutions, including community colleges, junior colleges and vocational-technical schools.

Acronyms

AAAS: American Association for the Advancement of Science
AACC: American Association of Community Colleges
AMATYC: American Mathematical Association of Two-Year Colleges
AMS: American Mathematical Society
ASA: American Statistical Association
ASAUGW: American Statistical Association Undergraduate Guidelines Workgroup
AAC&U: Association of American Colleges and Universities
AAU: Association of American Universities
APLU: Association of Public and Land-grant Universities
CSPCC: Characteristics of Successful Programs in College Calculus
CUPM: Committee on the Undergraduate Program in Mathematics
CBMS: Conference Board of the Mathematical Sciences
CRAFTY: Curriculum Renewal Across the First Two Years
HBCU: Historically Black Colleges and Universities
HHMI: Howard Hughes Medical Institute
ISSUES: Integration of Strategies that Support Undergraduate Education in STEM
INGenIOuS: Investing in the Next Generation through Innovative and Outstanding Strategies
JMM: Joint Mathematics Meeting
MOOC: Massive Open Online Course
MAA: Mathematical Association of America
NCTM: National Council of Teachers of Mathematics
NSF: National Science Foundation
NMP: New Mathways Project
PCAST: President's Council of Advisors on Science and Technology
Project ACCCESS: Advancing Community College Careers: Education, Scholarship, and Service
Project NExT: New Experiences in Teaching
STEM: Science, Technology, Engineering, and Mathematics
SIAM: Society for Industrial and Applied Mathematics

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Appendix A

Common Vision 2025 Conference Working Group

This project included a two-and-a-half-day workshop held in May 2015 at ASA headquarters in the Washington, D.C. area. Participants represented all of the professional associations that have as one of their primary objectives the increase or diffusion of knowledge in one or more of the mathematical sciences, as well partner disciplines in science, technology, and engineering. We were fortunate to engage participants from outside the academy, from higher education advocacy organizations, and also from industry. Much of the work for the forward-looking portions of this report was done at the workshop, and we appreciate the expertise and enthusiasm of the workshop participants (listed below) who did this work. When working within such a diverse group, there can be communication challenges; e.g., "pathways" can mean different things depending on institutional context. We are especially grateful that everyone was so willing to approach the work with an open mind and a sense of humor.

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Jesus DeLoera, *University of California at Davis*
Su Doree, *Augsburg College*
Rob Farinelli, *College of Southern Maryland*
Katie Fowler, *Clarkson University*

Jim Gates, *University of Maryland*
Howard Gobstein, *Association of Public and Land-grant Universities*
Mark Green, *University of California, Los Angeles*
Charles Henderson, *Western Michigan University*
Tim Hesterberg, *Google*
Bob Hilborn, *American Association of Physics Teachers*
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Jeff Humpherys, *Brigham Young University*
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Rob Kimball, *Wake Technical Community College*
Eric Kostelich, *Arizona State University*
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Suzanne Lenhart, *University of Tennessee*
Rachel Levy, *Harvey Mudd College*
Jim Lewis, *National Science Foundation*
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Uri Treisman, *University of Texas*
Peter Turner, *Clarkson University*
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Appendix B

The Seven Curricular Guides

The following seven guides have together formed the basis of our analysis in Chapter 2.

AMATYC Guide: *Beyond Crossroads*

In 2006, the American Mathematical Association of Two-Year Colleges (AMATYC) released *Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College.* This document was an update of the 1995 publication *Crossroads in Mathematics: Standard for Introductory College Mathematics* and reflects the contributions of hundreds of individuals. Through numerous fora and working sessions at annual conferences, focus sessions at affiliate conferences, and online questionnaires, the entire AMATYC membership as well as members of other professional organizations had multiple opportunities to provide feedback and input. The following is the Executive Summary, taken verbatim from the *Beyond Crossroads* website (beyondcrossroads.matyc.org/doc/ES.html).

From *Beyond Crossroads*

Background

As part of its mission to promote and improve mathematics education in two-year colleges, AMATYC published its first standards document, *Crossroads in Mathematics: Standards for Introductory College Mathematics*, in 1995. This was the first publication to communicate principles and standards for the teaching and learning of mathematics in two-year colleges. It emphasized desired modes of student thinking and provided guidelines for selecting content and instructional strategies. These standards were widely accepted and are as important today as they were in 1995.

With the 2006 publication of *Beyond Crossroads*, AMATYC re-affirms and builds upon the principles and standards set forth in 1995. In its five Implementation Standards, *Beyond Crossroads* advocates for informed decision-making and outlines expectations, responsibilities, recommendations, and action items for students, faculty, departments, and institutions. *Beyond Crossroads* is a call for continuous improvement in content, pedagogy, and professionalism.

Changing Environments

Informed citizens in today's global society need to be quantitatively literate. Technological advances continue to influence both the mathematics that is used in the workplace and the career opportunities available to college students. Between now and 2012, the number of high-skilled jobs as a percentage of the workforce is expected to increase and the number of unskilled jobs is expected to decrease.

Further, the environment for learning and teaching mathematics in higher education continues to experience significant changes. Advances in technology affect the mathematics that should be taught and the pedagogy that should be used to teach it. These changes create new challenges for faculty development, recruitment, and preparation of replacement faculty.

Faculty, with support from their institutions, shoulder the day-to-day responsibility for responding to this changing environment. They must continue to grow in their mathematical and pedagogical knowledge, contribute to their profession, address the learning needs of their diverse students, and prepare quantitatively literate citizens for the future. Creating a learning environment that responds to our ever-changing technological society requires the active involvement of every faculty member, department, and institution.

Basic Principles and Implementation Standards

Mathematics courses taken during the first two years of college offer students an opportunity to acquire mathematical to use technology as a learning tool, and to improve their ability to solve problems. It is important that students develop a greater appreciation for mathematics and an increased confidence to use mathematics. Improving the undergraduate mathematical experience and increasing the number of graduates in science, technology, engineering, and mathematics programs is widely accepted as essential to our nation's ongoing vitality.

Basic Principles. The foundation of all the standards presented in Beyond Crossroads:

- **Assessment** The assessment of student learning in mathematics should be a fundamental tool for the improvement of instruction and student learning.
- **Broadening** Mathematics courses and programs in the first two years of college should broaden students' options in educational and career choices.
- **Equity and Access** All students should have equitable access to high-quality, challenging, effective mathematics instruction and support services.
- **Innovation** Mathematics programs should be thoughtfully constructed to approach content and instruction with appropriate use of traditional and innovative methods.
- **Inquiry** Effective mathematics instruction should require students to be active participants.
- **Quantitative Literacy** Quantitative literacy should be integrated throughout the mathematics program and the college curricula.

- **Relevance** The mathematics that students study should be meaningful and foster their appreciation of the discipline.
- **Research into Practice** The practice of mathematics teaching should be guided by research on teaching and learning.
- **Technology** Technology should be integral to the teaching and learning of mathematics.

Implementation Standards

Beyond Crossroads introduces five Implementation Standards that extend the Standards for Intellectual Development, Content, and Pedagogy presented in the 1995 *Crossroads in Mathematics*. These five standards, with accompanying implementation recommendations and action items, are intended to guide the decision making of professionals in selecting and putting strategies into practice to meet the challenges of improving student learning in mathematics.

Implementation Standards. Guidelines for faculty, departments, and institutions for improving mathematics education:

- **Student Learning and the Learning Environment** Mathematics faculty and their institutions will create an environment that optimizes the learning of mathematics for all students.
- **Assessment of Student Learning** Mathematics faculty will use the results from the ongoing assessment of student learning of mathematics to improve curricula, materials, and teaching methods.
- **Curriculum and Program Developmen** Mathematics departments will develop, implement, evaluate, assess, and revise courses, course sequences, and programs to help students attain a higher level of quantitative literacy and achieve their academic and career goals.
- **Instruction** Mathematics faculty will use a variety of instructional strategies that reflect the results of research to enhance student learning.
- **Professionalism** Institutions will hire qualified mathematics faculty, and these faculty will engage in ongoing professional development and service.

Embracing Change and Continuous Improvement

Implementing the standards outlined in *Beyond Crossroads* includes making a commitment to continuous improvement in instruction, student learning, and professional development. Improvement may involve changing actions and philosophies of faculty, departments, and institutions. The Beyond Crossroads Implementation Cycle is a process for continuous improvement used to assess and improve any activity or project. AMATYC advocates the use of the principles and standards of *Beyond Crossroads* for guiding discussions and decision making about the direction of change in mathematics education.

Embracing the call to continuous improvement is a component of professional growth. Indeed, the challenges of making and embracing change are numerous. For some professionals, implementing change is energizing. For others, any change can be challenging. Embracing change in mathematics education should be a thoughtful process of planning, implementing, evaluating, and documenting, followed by redefining, implementing again, and improving the action(s) in the future to enhance student learning in mathematics. Continuous improvement in mathematics instruction is essential to improving student learning.

Moving from Vision to Reality

Beyond Crossroads sets forth ambitious goals and outlines key implementation roles for institutions, departments, faculty, students, and other stakeholders to reach these goals. Mathematics faculty should take the lead but will need the active cooperation of the entire mathematics community to implement changes to improve their teaching, student outcomes, and the mathematical abilities of future citizens. To move from vision to reality, faculty, departments, institutions, and students need to respond to each of their respective accountabilities:

Faculty

- accept responsibility for continuing to learn about mathematics and effective mathematics instruction
- use a multifaceted approach to instruction, not only to address a variety of learning styles, but also to reveal the richness and interconnectedness of mathematics
- determine desirable quantitative literacy outcomes for each mathematics course and promote quantitative literacy as an appropriate general education outcome.

Mathematics Departments

- incorporate workplace skills into all programs
- seek input from business and industry and require students to solve realistic problems with technology available in the workplace
- place K–12 teacher recruitment and preparation as high priorities of the department and cooperate with four-year institutions to provide supervised field experiences for future teachers.

Higher Education Institutions

- recruit and hire qualified mathematics faculty and then provide them with mentoring and ongoing professional development
- promote quantitative literacy across the curriculum and in general education courses
- cooperate with business and industry to collect information about the skills and knowledge that their employees need and use this information when assessing courses and programs.

Students

- accept responsibility for their learning and seek the resources that can help them attain their academic goals

- take their mathematics courses early in their program and set high mathematical expectations for themselves
- examine mathematical concepts from multiple perspectives and employ a variety of problem-solving strategies
- communicate mathematics, both orally and in writing.

Standards for Intellectual Development, Content, and Pedagogy

The 1995 *Crossroads in Mathematics* presented the three sets of standards listed below. Putting all these standards into practice has a greater effect on the improvement of student learning and professionalism of faculty than implementing any single standard in isolation.

Standards for Intellectual Development. Guidelines for desired modes of student thinking and goals for student outcomes:

- Problem solving
- Modeling
- Reasoning
- Connecting with other disciplines
- Communicating
- Using technology
- Developing mathematical power
- Linking multiple representations

Standards for Content. Guidelines for the selection of content in courses and programs:

- Number sense
- Symbolism and algebra
- Geometry and measurement
- Function sense
- Continuous and discrete models
- Data analysis, statistics, and probability
- Deductive proof

Standards for Pedagogy. Guidelines for instructional strategies in active student learning:

- Teaching with technology
- Active and interactive learning
- Making connections
- Using multiple strategies
- Experiencing mathematics

Roles of Stakeholders

The complex issues surrounding learning, assessment, curriculum, teaching, and professionalism can only be fully addressed through actions of all stakeholders, including leaders of K–16 education, business and industry, publishers, government, and society. Thoughtful collaboration among the stakeholders can produce greater results than any action taken alone. Faculty and institutions cannot accomplish their goals for improving mathematics education alone — the principles and standards in *Beyond Crossroads* are best served when two-year college mathematics faculty and institutions collaborate with the following entities:

- the broader mathematics community to build public understanding of and support for improvements in mathematics education
- K–12 school districts and four-year institutions to align exit and entrance requirements, instructional strategies, and curricula
- publishers and developers of instructional materials to create standards-based instructional resources
- faculty in other disciplines to infuse mathematics across the curriculum
- business and industry so that desired employee skills and strategies for achieving them are outlined and incorporated into mathematics courses and programs
- professional societies, government agencies, and educational institutions to build consensus and provide guidance to practitioners.

ASA Guides

The American Statistical Association (ASA) has published undergraduate curricula guides alone and in collaboration with the other associations. There are two key documents we examined as part of the *Common Vision* project, *Guidelines for Undergraduate Programs in Statistical Science* and *Guidelines for Assessment and Instruction in Statistics Education College Report*.

Guidelines for Undergraduate Programs in Statistical Science

The 2014 *Guidelines for Undergraduate Programs in Statistical Science* reflect significant changes in curriculum and recommended pedagogy since the ASA's previous guidelines were disseminated in 2000. These are endorsed by the ASA Board of Directors.

Key points include:

> *Increased importance of data science*. Working with data requires extensive computing skills. To be prepared for statistics and data science careers, students need facility with professional statistical analysis software, the ability to access and manipulate data in various ways, and the ability to perform algorithmic problem-solving. In addition to more traditional mathematical and statistical skills, students should be fluent in higher-level programming languages and facile with database systems.

Real applications. Data should be a major component of statistics courses. Programs should emphasize concepts and approaches for working with complex data and provide experiences in designing studies and analyzing non-textbook data.

More diverse models and approaches. Students require exposure to and practice with a variety of predictive and explanatory models in addition to methods for model building and assessment. They must be able to understand issues of design, confounding, and bias. They need to know how to apply their knowledge of theoretical foundations to the sound analysis of data.

Ability to communicate. Students need the ability to communicate complex statistical methods in basic terms to managers and other audiences and to present results in an accessible manner. They must have a clear understanding of ethical standards. Programs should provide multiple opportunities to practice and refine these statistical practice skills.

These guidelines are intended to be flexible while ensuring that programs provide students with the appropriate background along with critical thinking and problem-solving skills necessary to thrive in our increasingly data-centric world. They encourage creativity within programs toward the goal of providing a synthesis of theory, methods, computation, and applications.

Guidelines for Assessment and Instruction in Statistics Education College Report

The 2005 *Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report* is the ASA's report on introductory statistics curricula. The following is its executive summary (verbatim):

From *Guidelines for Assessment and Instruction in Statistics Education*

Executive Summary

The American Statistical Association (ASA) funded the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Project, which consists of two groups, one focused on K–12 education and one focused on introductory college courses. This report presents the recommendations developed by the college group. The report includes a brief history of the introductory college course and summarizes the 1992 report by George Cobb that has been considered a generally accepted set of recommendations for teaching these courses. Results of a survey on the teaching of introductory courses are summarized, along with a description of current versions of introductory statistics courses. We then offer a list of goals for students, based on what it means to be statistically literate. We present six recommendations for the teaching of introductory statistics that build on the previous recommendations from Cobb's report. Our six recommendations include the following:

1. Emphasize statistical literacy and develop statistical thinking

2. Use real data
3. Stress conceptual understanding, rather than mere knowledge of procedures
4. Foster active learning in the classroom
5. Use technology for developing conceptual understanding and analyzing data
6. Use assessments to improve and evaluate student learning

The report concludes with suggestions for how to make these changes and includes numerous examples in the appendices to illustrate details of the recommendations.

MAA Guides

The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) is charged with making recommendations to guide mathematics departments designing curricula for their undergraduate students. CUPM began issuing guidelines in 1953, updating them at roughly 10-year intervals. The most recent guide appeared in 2015. Curriculum Renewal Across the First Two Years (CRAFTY) is a subcommittee of the Committee on the Undergraduate Program in Mathematics (CUPM) and is charged with monitoring ongoing developments in curricula for the first two years of college mathematics with the intention of eventually making general recommendations. The CUPM and CRAFTY reports parallel each other, and therefore we treat them together in this section.

CUPM

The most recent CUPM *Curriculum Guide to Majors in the Mathematical Sciences* (2015) reaffirms the principles in the 2004 report and offers some important thoughts on the needs of incoming math majors:

> As mathematics grows in depth and applicability, effective mathematical education becomes more important than ever. At the same time, this broader scope of our discipline works against a single shared understanding of the phrase "mathematics major." The mathematical preparation of beginning college students may also change if the Common Core State Standards are implemented as envisioned. Against this background it makes little sense to prescribe in detail a single static list of "essential" requirements for a major in the mathematical sciences; many reasonable possibilities exist.

In addition to this observation that effective implementation of the Common Core State Standards will significantly change the preparation of incoming college students and will necessitate changes in the preparation of pre-service teachers, the guide calls on mathematics departments to prepare students for the modern workforce.

CUPM gives general principles and several recommendations.

From *CUPM 2015*

General Principles

Major programs in the mathematical sciences should ensure that all students come to see mathematics as an engaging field, rich in beauty, with powerful applications to other subjects and to contemporary open questions.

Each department should create and maintain a community that welcomes and supports all students, including those from groups that have been traditionally underrepresented in mathematics.

Recommendations

- A healthy mathematical sciences program should incorporate intentional evolution and continual improvement.
- Every mathematical sciences department should have and follow a strategic plan that acknowledges local conditions and resources, but is also informed by recommendations from the larger mathematical community.
- Planning and renewal should be guided by consultation both within the department and with outside stakeholders.
- Departments should assess their progress in meeting cognitive and content goals through systematic collection and evaluation of evidence.

Cognitive Goals

1. Students should develop effective thinking and communication skills.
2. Students should learn to link applications and theory.
3. Students should learn to use technological tools.
4. Students should develop mathematical independence and experience open-ended inquiry.

Content Recommendations

1. Mathematical sciences major programs should include concepts and methods from calculus and linear algebra.
2. Students majoring in the mathematical sciences should learn to read, understand, analyze, and produce proofs, at increasing depth as they progress through a major.
3. Mathematical sciences major programs should include concepts and methods from data analysis, computing, and mathematical modeling.
4. Mathematical sciences major programs should present key ideas and concepts from a variety of perspectives to demonstrate the breadth of mathematics.
5. All students majoring in the mathematical sciences should experience mathematics from the perspective of another discipline.
6. Mathematical sciences major programs should present key ideas from complementary points of view: continuous and discrete; algebraic and geometric; deterministic and stochastic; exact and approximate.

7. Mathematical sciences major programs should require the study of at least one mathematical area in depth, with a sequence of upper-level courses.
8. Students majoring in the mathematical sciences should work, independently or in a small group, on a substantial mathematical project that involves techniques and concepts beyond the typical content of a single course.
9. Mathematical sciences major programs should offer their students an orientation to careers in mathematics.

Below is the executive summary of the 2004 guide which details a list of recommendations and suggests ways that departments can evaluate its progress in meeting them.

From *Executive Summary of CUPM 2004*

The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) is charged with making recommendations to guide mathematics departments in designing curricula for their undergraduate students. CUPM began issuing reports in 1953, updating them at roughly 10-year intervals. *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004* is based on four years of work, including extensive consultation with mathematicians and members of partner disciplines.

Available at www.maa.org/cupm/, *CUPM Guide 2004* contains the recommendations unanimously approved by CUPM in January 2003.

Many recommendations in *CUPM Guide 2004* echo those in previous CUPM reports, but some are new. In particular, previous reports focused on the undergraduate program for mathematics majors, although with a steadily broadening definition of the major. *CUPM Guide 2004* addresses the *entire* college-level mathematics curriculum, for *all* students, even those who take just one course. *CUPM Guide 2004* is based on six fundamental recommendations for departments, programs and all courses in the mathematical sciences. The MAA Board of Governors approved these six recommendations at their Mathfest 2003 meeting.

Recommendation 1: *Mathematical sciences departments should*

- *Understand the strengths, weaknesses, career plans, fields of study, and aspirations of the students enrolled in mathematics courses;*
- *Determine the extent to which the goals of courses and programs offered are aligned with the needs of students as well as the extent to which these goals are achieved;*
- *Continually strengthen courses and programs to better align with student needs, and assess the effectiveness of such efforts.*

Recommendation 2: *Every course should incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. More specifically, these activities should be designed to advance and measure students' progress in learning to*

- *State problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently;*
- *Approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures;*
- *Read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking.*

Recommendation 3: *Every course should strive to*

- *Present key ideas and concepts from a variety of perspectives;*
- *Employ a broad range of examples and applications to motivate and illustrate the material;*
- *Promote awareness of connections to other subjects (both in and out of the mathematical sciences) and strengthen each student's ability to apply the course material to these subjects;*
- *Introduce contemporary topics from the mathematical sciences and their applications, and enhance student perceptions of the vitality and importance of mathematics in the modern world.*

Recommendation 4: *Mathematical sciences departments should encourage and support faculty collaboration with colleagues from other departments to modify and develop mathematics courses, create joint or cooperative majors, devise undergraduate research projects, and possibly team-teach courses or units within courses.*

Recommendation 5: *At every level of the curriculum, some courses should incorporate activities that will help all students progress in learning to use technology*

- *Appropriately and effectively as a tool for solving problems;*
- *As an aid to understanding mathematical ideas.*

Recommendation 6: *Mathematical sciences departments and institutional administrators should encourage, support and reward faculty efforts to improve the efficacy of teaching and strengthen curricula.*

Part I of CUPM Guide 2004 elaborates on the recommendations given therein and suggests ways that a department can evaluate its progress in meeting them. Part II contains supplementary recommendations concerning various types of students:

A. Students taking general education or introductory courses in the mathematical sciences;

B. Students majoring in partner disciplines, including those preparing to teach mathematics in elementary or middle school;

C. Students majoring in the mathematical sciences;

D. Mathematical sciences majors with specific career goals: secondary school teaching, entering the non-academic workforce, and preparing for post-baccalaureate study in the mathematical sciences and allied disciplines.

Specific methods for implementation are not prescribed, but the online document *Illustrative Resources for CUPM Guide 2004* at www.maa.org/cupm/ describes a variety of experiences and resources associated with these recommendations. These illustrative examples are not endorsed by CUPM, but they may serve as a starting point for departments considering changes to their programs. Pointers to additional resources, such as websites (with active links) and publications, are also given.

The 2004 guide also states, "It is more important that mathematics majors study statistics or probability with an approach that is data-driven than one that is calculus-based. The concepts and reasoning needed in a statistics course is different from those needed in mathematics courses. An introductory course should (1) emphasize statistical thinking: the importance of data production; the omnipresence of variability; and the quantification and explanation of variability; (2) place a strong emphasis on data and concepts and less emphasis on theory and recipes; and (3) foster active learning." (p. 48).

CRAFTY

The most recent work of CRAFTY has focused on exploring the mathematical needs of partner disciplines and on the college algebra course. The *CUPM Curriculum Guide 2004* called attention to the fact that, "Unfortunately, there is often a serious mismatch between the original rationale for a college algebra requirement and the actual needs of the students who take the course. A critically important task for mathematical sciences departments at institutions with college algebra requirements is to clarify the rationale for the requirements, determine the needs of the students who take college algebra, and ensure that the department's courses are aligned with these findings." In parallel with and in response to this charge from its parent committee, CRAFTY focused its work on two related areas:

- Determining the mathematical needs and priorities of our partner disciplines by convening a total of 22 weekend workshops of representatives of these disciplines.
- Supporting efforts of mathematics departments to develop and offer an engaging and appropriate college algebra courses.

These efforts culminated in two publications: *The Curriculum Foundations Project: Voices of the Partner Disciplines* (Ganter & Barker, 2004) and *Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra* (Ganter & Haver, 2011).

The first includes a list of recommendations as well as 18 individual disciplinary reports (Biology; Business and Management; Chemistry; Computer Science; Chemical Engineering; Civil Engineering; Electrical Engineering; Mechanical Engineering; Health-related Life Sciences; Interdisciplinary Core Mathematics; Mathematics; Physics; Statistics; Biotechnology and Environmental Technology; Electronics, Telecommunications and Semiconductor Technology; Information Technology; Mechanical and Manufacturing Technology; and K–12 Mathematics Teacher Preparation).

Workshops held for writing the first resulted in a set of recommendations for the first two years of undergraduate mathematics instruction. These recommendations are listed below.

From *Summary Recommendations: A Collective Vision*

Understanding, Skills, and Problem Solving

- Emphasize conceptual understanding.
- Emphasize problem solving skills.
- Emphasize mathematical modeling.
- Emphasize communication skills.
- Emphasize balance between perspectives.

Priorities for Content, Topics, and Courses

- Strive for depth over breadth. Explore locally what topics can be omitted and teach the remaining topics in more depth.
- Offer non-calculus-based descriptive statistics and data analysis in the first two years (either as a separate course or integrated into other courses).
- Offer discrete mathematics and mathematical reasoning in the first two years. Do not have calculus as a prerequisite.
- Continue to offer calculus and linear algebra in the first two years, but make the curriculum more appropriate for the needs of the partner disciplines.
- Replace traditional college algebra courses with courses stressing problem solving, mathematical modeling, descriptive statistics, and applications in the appropriate technical areas. Deemphasize intricate algebraic manipulation.
- Emphasize two- and three-dimensional topics.
- Pay attention to units, scaling, and dimensional analysis.

Instructional Techniques and Technology

- Use a variety of teaching methods … In particular, encourage the use of active learning.
- Improve interdisciplinary cooperation.
- Emphasize the use of appropriate technology.
- Emphasize the use of appropriate assessment.

Recommendations for Departments and the Profession

- Promote professional development.
- Establish mechanisms for the developent, review, and dissemination of effective instructional materials and techniques, including collaborative efforts between mathematicians and partner disciplines that result in innovative instructional materials.
- Encourage institutional assessment of programmatic changes.

The second volume begins with reports from participants in ve disciplinary workshops (Agriculture, Arts, Economics, Meteorology, and Social Science) and a summary of the recommendations of the overall Curriculum Foundations project.

From *Partner Discipline Recommendations for Introductory College Mathematics and the Implications for College Algebra*

A close examination of the first series of workshops found that the stereotype that users of mathematics care primarily about computational and manipulative skills, forcing mathematicians to cram courses full of algorithms and calculations, "is largely false" (Ganter and Barker, 2004). Indeed, disciplinary representatives "were unanimous in their emphasis on the overriding need to develop in students a conceptual understanding of the basic mathematical tools." The disciplinary reports provide guidance on the fundamental skills required for each discipline and emphasize that the abilities most valued are problem solving skills, mathematical modeling, communication skills, and command of appropriate technology. The workshops conducted during phase two of the project with disciplinary representatives from the social sciences and humanities, reconfirmed these universal needs in spite of the vast differences between the disciplines represented. Specifically, the five disciplinary reports in this volume indicate a need for mathematics courses that emphasize:

- Conceptual understanding and problem solving — communicating solutions to diverse audiences; precise and correct use of mathematics in presentations and reports
- Arithmetic and basic mathematical equations — relationships between variables; percentages, proportion, and measurement; translation of words into appropriate formulas and equations; graphical representations; unit conversions
- Problems in context — building analytical models and testing their viability; applying theory to real problems and evaluating alternative solutions; communicating and coordinating with disciplinary faculty to develop alternative problems; using context to inspire and create a need for mathematics (i.e., mathematics as a common technical language)
- Estimation and approximation — use of experimentation and exploration to discover mathematical concepts
- Statistics and quantitative data — measures of central tendency and standard deviation; analyzing data to make inferences and draw conclusions; presenting data as pictures (such as bar graphs, line graphs, and scatter plots)
- Appropriate use of technology — spreadsheets; geometrical/graphical software

The CRAFTY workshops engaged stakeholders from a large and diverse set of disciplines that are making use of mathematics. From *The Curriculum Foundations Project: Voices of the Partner Disciplines*:

> Students do not see the connections between mathematics and their chosen disciplines; instead, they leave mathematics courses with a set of skills that they are un-

> able to apply in non-routine settings and they do not appreciate the importance of these skills to their future careers. Indeed, the mathematics many students are taught often is not the most relevant to their chosen fields. For these reasons, faculty members outside mathematics often perceive the mathematics community as uninterested in the needs of non-mathematics majors, especially those in introductory courses.
>
> The mathematics community ignores this situation at its own peril since approximately 95% of the students in first-year mathematics courses go on to major in other disciplines. The challenge, therefore, is to provide mathematical experiences that are true to the spirit of mathematics yet also relevant to students' futures in other fields. The question then is not *whether* they need mathematics, but *what* mathematics is needed and in what context (p.1).

No mathematics department can possibly offer a different mathematics course for majors in each of the different disciplines represented on its campus, making the need to rethink and revise the most popular introductory mathematics courses such as college algebra even more critical. And since the broad categories of conceptual understanding, problem solving, mathematical modeling, and communication cut across the recommendations from all the partner disciplines, it makes sense to redevelop college algebra in a way that addresses these universal needs.

The second report continues with the college algebra guidelines developed by CRAFTY and endorsed by CUPM, reports from an NSF supported college algebra project, and papers describing the results of efforts led by four different members of CRAFTY to improve college algebra, three at their respective institutions and one with a consortium of historically black colleges and universities (HBCUs). Finally, the report outlines a set of recommendations for departments that are considering revitalizing college algebra are outlined. The college algebra guidelines appear below.

College Algebra Guidelines

These guidelines represent the recommendations of the MAA/CUPM subcommittee, Curriculum Renewal Across the First Two Years, concerning the nature of the college algebra course that can serve as a terminal course as well as a pre-requisite to courses such as pre-calculus, statistics, business calculus, finite mathematics, and mathematics for elementary education majors. They were endorsed on January 31, 2007 by CUPM, the Committee on the Undergraduate Program in Mathematics.

Fundamental Experience

College Algebra provides students a college level academic experience that emphasizes the use of algebra and functions in problem solving and modeling, provides a foundation in quantitative literacy, supplies the algebra and other mathematics needed in partner disciplines, and helps meet quantitative needs in, and outside of, academia. Students address problems presented as real world situations by creating and interpreting mathematical models. Solutions to the problems are formulated, validated, and analyzed using mental, paper and pencil, algebraic, and technology-based techniques as appropriate.

Course Goals

- Involve students in a meaningful and positive, intellectually engaging, mathematical experience.
- Provide students with opportunities to analyze, synthesize, and work collaboratively on explorations and reports.
- Develop students' logical reasoning skills needed by informed and productive citizens.
- Strengthen students' algebraic and quantitative abilities useful in the study of other disciplines.
- Develop students' mastery of those algebraic techniques necessary for problem-solving and mathematical modeling.
- Improve students' ability to communicate mathematical ideas clearly in oral and written form.
- Develop students' competence and confidence in their problem-solving ability.
- Develop students' ability to use technology for understanding and doing mathematics.
- Enable and encourage students to take additional coursework in the mathematical sciences.

Competencies

1. Problem Solving

   Goals for students include

   - solving problems presented in the context of real world situations with emphasis on model creation and interpretation;
   - developing a personal framework of problem solving techniques (e.g., read the problem at least twice; define variables; sketch and label a diagram; list what is given; restate the question asked; identify variables and parameters; use analytical, numerical and graphical solution methods as appropriate; determine plausibility of and interpret solutions);
   - creating, interpreting, and revising models and solutions of problems.

2. Functions and Equations

   Goals for the students include

   - understanding the concepts of function and rate of change;
   - effectively using multiple perspectives (symbolic, numeric, graphic, and verbal) to explore elementary functions;
   - investigating linear, exponential, power, polynomial, logarithmic, and periodic functions, as appropriate;
   - recognizing and using standard transformations such as translations and dilations with graphs of elementary functions;

- using systems of equations to model real world situations;
- solving systems of equations using a variety of methods;
- mastering algebraic techniques and manipulations necessary for problem-solving and modeling in this course.

3. Data Analysis

   Goals for the students include

   - collecting (in scientific discovery or activities, or from the Internet, textbooks, or periodicals), displaying, summarizing, and interpreting data in various forms;
   - applying algebraic transformations to linearize data for analysis;
   - fitting an appropriate curve to a scatter plot and use the resulting function for prediction and analysis;
   - determining the appropriateness of a model via scientific reasoning.

Emphasis in Pedagogy

Goals for the instructor include

- facilitating the development of students' competence and confidence in their problem-solving abilities;
- utilizing and developing algebraic techniques as needed in the context of solving problems;
- emphasizing the development of conceptual understanding of the mathematics by the students
- facilitating the improvement of students' written and oral mathematical communication skills;
- providing a classroom atmosphere that is conducive to exploratory learning, risk-taking, and perseverance;
- providing student-centered, activity-based instruction, including small group activities and projects;
- using technology (computer, calculator, spreadsheet, computer algebra system) appropriately as a tool in problem-solving and exploration;
- conducting ongoing assessment activities designed to determine when mid-course adjustments are warranted.

Assessment

- Assessment tools will measure students' attainment of course competencies, including:
  - solving problems and interpreting results using algebraic tools;
  - building and interpreting models and predicting results;
  - communicating processes and solutions orally and in writing;

  - making quantitative and algebraic arguments;
  - reading and interpreting data presented in various forms.
- Assessment tools will include
  - individual quizzes;
  - individual examinations;
  - additional activities or assignments, such as
    * individual or group homework, projects, and activities;
    * individual or group oral presentations;
    * portfolios that demonstrate student growth;
    * group quizzes and exams.
- The course will be assessed by analyzing its effectiveness in:
  - facilitating student achievement of the course competencies;
  - positively affecting student attitudes about mathematics;
  - preparing students for subsequent courses in mathematics and mathematics-dependent disciplines;
  - preparing students for subsequent endeavors in and outside academia.

SIAM Guides

Modeling Across the Curriculum

SIAM regularly writes curricular guides and makes further recommendations on education policy and curricula in areas relevant to applied and computational mathematics. The executive summary of the first *Modeling Across the Curriculum* guide appears below. The second Modeling Across the Curriculum workshop was held in January 2014, and the report came out later that year.

From *Modeling Across the Curriculum*

Executive Summary. *Where do we go from here?*

The Modeling Across the Curriculum workshop had three themes for discussion by professionals who are immersed in the areas:

- develop STEM high school courses based on math modeling,
- increase math modeling in undergraduate curricula, and
- assess/improve college STEM readiness.

There are many overlaps in the issues and the possible ways to address these challenges. Not surprisingly, the deeper we dig into these issues, the more there is a need to expand the view and take more aspects into account when proposing solutions. Education, with the resulting ability to think, process, use and understand information, forms a multi-layered set of issues. The complete summaries of workshop discussions are included in the report.

In this executive summary, we attempt to prioritize some specific objectives. The focus is on what SIAM, and its membership, can do in collaboration with other applied mathematicians and computational scientists.

The **top level goal** of proposed efforts is to:

ENGAGE AND KEEP YOUNG PEOPLE IN STEM DISCIPLINES, FROM K-12 THROUGH UNDERGRADUATE (AND GRADUATE) STUDIES, AND INTO THE WORKFORCE.

<u>Recommended Actions:</u>

I. Expand modeling in K–12:

   A. Identify one or two strong approaches to modeling curriculum for K–12 (possibly from schools with demonstrated strength based on success in SIAM's M3 Challenge, a national math modeling contest).

   B. Develop content and teacher training materials — how to do modeling; how to use models.

   C. Engage a network of experts for mentoring and inquiries. Involve high school teachers and judges with successful experience in SIAM's Moody's Mega Math Challenge as team coaches and evaluators.

   D. Build an awareness campaign for teachers, math curriculum supervisors, and academic counselors.

   E. Measure outcomes. Establish benchmarks of undergrad STEM retention from graduates of schools with strong modeling background vs those with standard course sequence in math. Monitor schools with pilot programs for change.

II. Develop a high school one semester or one year modeling course (with stratified content):

   A. Should be multi-disciplinary drawing together mathematical and scientific experiences.

   B. Should be influenced heavily by successful programs already offered.

   C. Develop a professional development training component for teachers to instruct and engage students on the modeling approach to education.

   D. An aspirational goal: make such a course MANDATORY for high school graduation in order to demonstrate the usefulness and relevance of other math courses, laying the foundation for success in college STEM majors.

III. Develop modeling-based undergraduate curricula

   A. Concentrate initially on the first year of undergraduate STEM experience

   B. Investigate two models:

      1. Using modeling and applications as a skeleton on which the calculus sequence is built, "Modeling across the Curriculum", or

      2. A first year modeling/applied mathematics course that precedes and motivates the study of calculus and other fundamental mathematics for STEM majors

C. Provide seed grants for faculty to develop, implement and evaluate new approaches to the high school — college math transition for STEM majors

IV. Develop a repository of materials for all aspects and levels of math modeling instruction and understanding. To include but not limited to: course lesson plans, articles, books, web sites, videos, contests, problems and solutions.

This process should be started immediately with a follow up workshop to address some of these specifics as early as possible, perhaps even during the summer of 2013. If that timing is feasible, then an aggressive timetable of important targets for the K–12 program would be:

Summer 2013 — Follow up workshop to zero in on the specific actions and related tasks. Lay groundwork for principle staff to move forward with actions.

Fall 2013 — Identify and begin discussions and capturing of information from high performing schools in modeling. Identify possible schools to host pilot programs — public and private, urban, suburban, rural.

Spring 2014 — Develop pilot curriculum in detail.

Summer 2014 — Train pilot school teachers to teach.

Fall 2015 — Pilot schools offer course. Mentors are available for help. Circle of feedback and encouragement. Establish relationship with all students for longitudinal study.

Spring 2016 — Gather feedback; adjust curriculum, and iterate the loop expanding the range of participating schools over the next several years.

A similar schedule would be applied to the undergraduate curriculum development. The evaluation would start with baseline data collection and then assess the changes as they are introduced, continuing with longitudinal studies.

The workshop plan predated the President's Council of Advisors on Science and Technology Engage to Excel report by several months but clearly begins to address some of the issues raised in a very constructive manner.

Undergraduate Degree Programs in Applied Mathematics

This report is meant to provide information for people interested in developing a new program in industrial or applied mathematics or enriching an existing one. It concludes by noting that new trends in industrial and applied mathematics, which include increasing focus on computational mathematics and interdisciplinary connections between STEM fields, would be interesting areas to investigate in a future report.

From *Undergraduate Degree Programs in Applied Mathematics*

Executive Summary.
SIAM Education Committee
Report on Undergraduate Degree Programs in Applied Mathematics, May 2014

With a global focus on preparing students to enter STEM (Science, Technology, Engineering and Mathematics) fields, undergraduate programs in Applied Mathematics have an important role to play in preparing the future workforce. The purpose of the SIAM advisory report was to describe the components of existing programs in Applied Mathematics. The intended audience includes people who may be interested in initiating new programs, improving existing programs, or policy-makers.

Components of an Undergraduate Applied Mathematics Program

To create typical profiles of undergraduate applied mathematics programs, we developed and administered a set of survey questions to a small set of colleges and universities varying in geographical region, size and focus. The survey, administered as a Google form, had two parts: program development, resources and challenges and curricular data. We gathered data from 12 institutions and summarized the results.

Characteristics of applied math programs

- Heavier requirements in numerical analysis, computer science, and physical sciences; balance of compulsory theoretical courses with real-world applied courses;
- flexibility in the program; specialized programs, for example in statistics, actuarial science, mathematical biology;
- research opportunities; industrial connections; interdisciplinarity;
- social activities, including lunches, math club, pi-mu-epsilon, SIAM student chapter; studies abroad
- capstones, including senior honors thesis or industrial experience
- summer research or industrial internships

Post-graduation outcomes

Major industries that seek applied math graduates include: financial, software/tech/internet, actuarial/insurance, aerospace, pharmaceutical, defense/government contractors, consulting, education, automobile/manufacturing, and oil and gas. The number of students seeking postgraduate education varied widely by program.

Challenges of Developing a New Program

- Resources to hire faculty to teach specialized courses
- Differentiating the program from existing majors
- Developing new courses
- Proving the demand and handling unexpected success
- Accreditation requirements
- Computer and software resources and support
- Creating and maintaining industrial connections

Typical Applied Mathematics Curriculum

	Major	Minor/Concentration
Required courses	Calculus sequence Differential equations Linear algebra Introduction to proofs Real analysis Numerical analysis Applied math or Modeling Probability Statistics Abstract algebra Introduction to programming Object-oriented languages	Calculus sequence Differential equations Linear algebra Introduction to programming
Recommended/Elective Courses	PDEs Math biology Operations research Complex analysis Discrete mathematics Geometry	Differential equations Linear algebra Discrete mathematics Numerical analysis Applied math or Modeling Operations research

Industrial Mathematics-related Skills

- Communication
- Ability to work in interdisciplinary environments
- Technical skills: math, computational and application

Undergraduate research-related skills

- Enhances student learning through mentoring relationships with faculty
- Increases retention
- Increases enrollment in graduate education and provides effective career preparation
- Develops critical thinking, creativity, problem solving and intellectual independence
- Develops an understanding of research methodology
- Promotes an innovation-oriented culture

Professional development at conferences

- Increase their breadth and depth of understanding of mathematics by attending talks.
- Network with students, faculty and industry representatives.
- Attend recruiting evenings and informational sessions for internships, REUs, jobs and graduate programs.

- Present posters and talks to practice mathematical communication.
- Experience being a part of the applied mathematics community.

The report also included information about student membership and other opportunities through professional organizations and connections to K–12 curriculum and teacher preparation.

Appendix C

Further Reports

In addition to the seven curricular guide that serve the central focus of this document, several other reports helped shape our thinking. These reports are either not curriculum guides per se, or are not focused on the undergraduate curriculum. There are some important topics we have intentionally not included, e.g., high school curriculum, developmental mathematics, masters and Ph.D. level education and curricula; they are beyond the scope of this initial phase.

The common themes identified in *Common Vision* are reflected in several other reports; full citations appear in the References section.

- *Transforming Post-Secondary Education in Mathematics: Report of a Meeting*
  www.tpsemath.org/
- *Towards Excellence: Leading a Mathematics Department in the 21st Century*
  www.ams.org/profession/leaders/workshops/towardsexcellence
- *A Vision: Mathematics for the Emerging Technologies*
  www.amatyc.org/?page=Publications
- INGenIOuS
  www.maa.org/ingenious
- ISSUES
  serc.carleton.edu/issues/index.html
- Qualifications for Teaching an Introductory Statistics Course
  magazine.amstat.org/blog/2014/04/01/asamaaguidelines/
- *Undergraduate Computational Science and Engineering Education*
  www.siam.org/about/pdf/CSE_Report.pdf